On Assessment of Teaching a Mathematical Topic using Neural Networks Model (with a Case Study)

H. M. Mustafa, Ayoub Al-Hamadi and Mohammed H. Kortam

Abstract—This paper presents an interdisciplinary approach adopting realistic and fairly performance assessment of two educational Computer-Assisted Learning (CAL) module(s). They concerned with learning "How to solve long division problem?". By sequential processes as: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). Both modules were designed carefully in order to develop two CAL multimedia packages with effective utilization of visual and/or auditory tutorial materials. Additionally, both modules were provided for teaching the above adopted mathematical topic applied to children with average age about 11 years old (at the fifth grade class level in elementary schools). Interestingly, designed CAL modules proved well agreement likewise the Artificial Neural Network (ANN) associative memories theories, cognitive multimedia, and classical conditioning.

Index Terms—Computer Assisted Learning, Associative memory, Multimedia Learning Theory, Neural Networks Modeling.

I. INTRODUCTION

THE field of the learning sciences is currently represented by a growing community internationally. Many educational experts now recognize that conventional ways of conceiving knowledge associated with educational systems performance as well as assessment of technology-mediated learning processes are facing increasingly challenges. That is due to rapid technological and social changes arise in this time considering modified educational field applications [1-3]. In U.S.A., it had been announced that last decade (1990-2000) named as Decade of the brain [4]. Therefore, a recent evolutionary trend has been adopted by educationalists based on modeling of main human brain functions (learning and memory).

Accordingly, educational experts have focused their attention on building up bridged interdisciplinary models incorporating Neuro-physiology, Educational Psychology, and Cognitive science. That bridged trend has been confirmed by the strong connections among three of the most prosperous research areas of Educational Psychology, Artificial Neural Networks (ANNs), and Cognitive Sciences [1-7]. Recently neural networks theorists have adopted realistic modeling of (ANNs) combined with neuroscience as a novel research direction in searching for optimal teaching children methodologies [5-7]. Hence, application of interdisciplinary multimedia packages of Computer-Assisted Learning (CAL) inspired by cognitive modeling of main human brain functions considered as a modified trend for conventional educational systems [4-9].

This piece of research belongs to a rather challenging interdisciplinary approach adopting realistic and fairly assessment of educational processes based on Computer-Assisted Learning (CAL) module(s). This trend is confirmed by the strong connections among three of the most prosperous research areas of Educational Psychology, Artificial Neural Networks, and Cognitive Sciences. Herein, this work presents comparative assessments for three different experimental educational methodologies. Presented assessments performed using a computer program for measuring children's response time as learning convergence time parameter. More precisely, timely practical based assessment processes (at classrooms) have been well supported by exploiting realistic Neural Networks modeling of learning time response. Accordingly, two modules were carefully designed aiming to develop CAL multimedia packages with effective utilization of visual and/or auditory tutorial materials. Furthermore, both modules are provided for teaching an adopted mathematical topic to children at the fifth grade class level (in elementary schools), with average age about 11 years old. Specifically, in presented paper, they were concerned with learning “How to solve long division problem?”. By sequential processes as: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). Finally, designed CAL modules proved to agree likewise the cognitive multimedia, classical conditioning, and associative memories theories.

More recently, a fairly unbiased approach has been presented for assessment of a mathematical learning topic considering children's scores (achievement marks) as a learning parameter [5-6]. Therein, presented ANN models provide classical (conventional) teaching approach with noticeable improvement of educational systems quality. This trend has been adopted to search for optimal analysis of Phonics Methodology superiority for teaching children how to read [8-12]. Herein, realistic and fairly model adopted for comparative assessments of educational improvement observed after running of some multimedia (CAL) module(s). Furthermore, the cognitive multimedia theory suggests that the visual and auditory material should be presented simultaneously -based upon memory association- to reinforce the retention of learned materials [7].

This cognitive theory adopted as an optimal approach for improving teaching/learning performance of a mathematical topic to children of about 11 years age. Interestingly, at that
age, elementary schools' children may be qualified to learn "basic building blocks" of cognition. This result supported by what has been recently declared by National Institutes of Health (NIH) in U.S.A considering children [1] [13]. The suggested mathematical topic is teaching children algorithmic process for performing solving of long division problem. That solution consisted of five sequentially steps as follows: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary) [14]. Specifically, two arbitrary integer numbers presenting dividend and divisor were chosen in a random manner. By detail, adopted principal algorithm for applied Computer Aided Learning (CAL) package For more details, an overview concerned with the effect of information technology computer (ITC) on mathematical education, it is advised referring to, [15]. Practically, obtained assessment results revealed that children subjected to a CAL module with simultaneously applied learning materials (visual and auditory), have better learning performance. I.e. their learning convergence to desired solution has been reached faster with less prone to errors; which conclusively indicates optimality of applied teaching methodology. In other words, Optimal teaching methodology of learned mathematical topic (long division sequential processes) could be reached after simultaneous application of visual and auditory learning That interesting conclusive remark agrees likewise the cognitive multimedia theory revealing that simultaneous application -in practical educational field- of visual and auditory tutorial materials is highly recommended [2-3] [7].

The rest of this paper is organized as follows. At next section, a basic interactive educational model illustrated by generalized block diagram are presented along with a realistic ANN learning / teaching model. Associative brain memory function (correlation matrix) is presented given at third section via revising of Pavlovian conditioning modeling [16]. That correlation matrix brain model recently applied for teaching children how to read using phonics methodology (function, correlation matrix) is presented is given at third section via revising of Pavlovian conditioning modeling [16]. Obtained simulation results as well as the practical results (at the case study) are given at the fourth section after application of suggested CAL package. At the fifth section, some interesting conclusions, suggestions for future work are presented. Finally, by the end of this research work; three appendices are attached: APPENDIX A, APPENDIX B, and APPENDIX C. They are concerned with print screens of assessment program and two flowcharts for applied CAL, and ANN simulation programs respectively.

II. BASIC EDUCATIONAL MODELLING

A. General Interactive Educational Model

Generally, practical performing of interactive learning processes -from neurophysiologic P.O.V. - utilizes two basic and essential brain cognitive functions. Both functions are required in performing efficiently learning / teaching interactive processes in accordance with behaviorism approach [18-21] as follows. Firstly, pattern classification /recognition function based on visual /audible interactive signals stimulated by CAL packages. Secondly, associative memory function is used which originally based on Pavlovian classical conditioning motivated by Hebbian learning rule (details are given at the third section). Referring to Fig.1, it illustrates teaching model that well qualified to perform simulation of brain cognitive functions mentioned in above. At this figure, inputs to the neural network learning model are provided by environmental stimuli (unsupervised learning).The correction signal, in case of learning with a teacher is given by responsive output action of the model. It would be provided to ANN model by either unsupervised learning signal (environmental conditions) or by teacher's supervision signal . Interestingly, tutor plays a role in improving the input data (stimulating learning pattern), by reducing noise and redundancy of model pattern input. According to tutor's experience concerned with either conventional (classical) learning or CAL; after analysis of obtained realistic simulation results, two practical issues were observed. Firstly, the model given at Fig.1 can be provided with cleared data by maximizing its signal to noise S/N ratio. Such better S/N ratio in noisy learning environment results in improvement of learning performance quality. In other words, by less number of training cycles, desired learning convergence could be attained (as shown at Table 1). Similarly, improvement of learning rate values results in better learning performance indicated by decreased learning response (convergence) time. That improvement illustrated well by simulation results presented graphically at Fig.2 and tabulated (on the average) at Table 2. Secondly, tutors' experience observed to be transferred via a link to children's brain model (ANN) as a correction simulating signal. So, that experience may be capable of increasing number of neurons contributing to learning process convergence. Specifically, those increased neurons are placed at hippocampus children brain area. Detailed analysis and evaluation concerned with the second issue is given at third section.

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![Diagram of Interactive Educational Model](image)

**Fig.1.** Illustrates a general view for interactive educational process, adapted from [5].

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<table>
<thead>
<tr>
<th>Noise Power in Learning Environment</th>
<th>0.2</th>
<th>0.1</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S/N) Ratio of Input Data</td>
<td>5</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Convergence Time (cycles)</td>
<td>85</td>
<td>62</td>
<td>47</td>
</tr>
</tbody>
</table>

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Table 1. Effect of noisy environment on learning convergence time adapted from [7]
A. Basic ANN Model

Searching for optimal learning/teaching methodology is inspired by realistic cognitive simulation and modeling of computer assisted learning (CAL) as well as classical teaching performance. By using relevant Artificial Neural Network (ANN) learning model, fairly assessments for suggested leaning/teaching topics have been performed. Consequently, optimal tutoring methodology could be reached after analysis and evaluation of obtained simulation assessment fair results. At Fig.2, a general block diagram for an ANN learning/teaching model is depicted. It presents realistic simulation of two diverse learning paradigms. Both concerned with interactive tutoring /learning process as well as self-organized learning. The first paradigm is concerned with classical (supervised by tutor) learning observed at our classrooms (face to face tutoring). Accordingly, this paradigm proceeds interactively via bidirectional communication process between a tutor and his learner(s) [18]. The second paradigm performs self-organized (unsupervised) tutoring process [22-24].

Referring to Fig.2, the error vector at any time instant \( t \) observed during learning processes are given by:

\[
\vec{e}(n) = \vec{y}(n) - \vec{d}(n)
\]

Where \( \vec{e}(n) \) is the error correcting signal controlling adaptively the learning process, \( \vec{x}(n) \) is the input stimulus, \( \vec{y}(n) \) is the output response vectors, and \( \vec{d}(n) \) is the desired numeric value(s).

The following equations are easily deduced:

Where X is input vector, W is the weight vector, \( \phi \) is an activation (odd sigmoid) function characterized by \( \lambda \) as gain factor and Y as its output. \( e_k \) is the error value, and \( d_k \) is the desired output. Noting that \( AW(k)(n) \) is the dynamical change of weight vector value connecting the k th and ith neurons. Eqs. (2-5) are commonly applied for both the supervised (interactive learning with a tutor), and the unsupervised (learning though students’ self- study) paradigms. The dynamical changes of weight vector value for supervised phase are given as following:

Where, \( \eta \) is the learning rate value during learning process. However, for unsupervised paradigm, the dynamical change of weight vector value is given by:

Noting that \( e_k(n) \) in (6) is substituted by \( y_k(n) \) at any arbitrary time instant (n) during learning process.

III. ASSOCIATIVE BRAIN MEMORY FUNCTION

A. Revising of Phonics Methodology

Analysis and evaluation of associative memory function is introduced through realistic simulation and modelling of teaching how to read processes [8][9][17]. Those three research papers inspired by Hebbian learning rule adopted after Pavlovian psycho-experimental modelling [16]. Nevertheless, recalling teaching how to read using phonics methodology is performed directly by coincident association (correlation matrix) between pronounced sound (phoneme) and its corresponding letter / word (as given at Fig.4). At that figure, an ANN model analogously presents Pavlovian conditioning as well as Hebbian learning rule obeying original self-organized paradigm and learning [22][24]. In some details, inputs to the model considered either conditioned or unconditioned stimuli. Therefore, visual and audible signals are provided interchangeably for training suggested model in order to get output desired learning responses (writing heard signal and pronouncing seen signal). Additionally, introduced model shown to obey mathematical analysis elaborated for general Hebbian algorithm by correlation matrix memory, Pavlovian behavioural learning process, and learning by interaction with environment [16] [22-24].
sound (heard) stimulus and visual (sight) stimulus respectively. The outputs O1, O2 represent pronouncing and image recognition processes respectively. In order to justify the superiority and optimality of phonic approach over other teaching to read methods, an elaborated mathematical representation is introduced for two different neuro-biologically based models. Any of the models needs to learn how to behave (to perform reading tasks). Somebody has to teach (for supervised learning) - not in our case - or rather for our learning process is carried out on the basis of former knowledge of environment problem (learning without a teacher).

B. Effect of increasing neurons' number on learning performance

1) Neurons' number effect on learning response time

In below Table 3 illustrates how increased number of neurons (contributing to learning process) results in better learning performance (minimizing learning response time). This time considered as correction elapse time needed under tutor's supervision. It is measured in total numbers of learning response (cycles). The obtained tabulated results are obtained after running of a realistic simulation computer program (its flowchart is given at APPENDIX A). Noting that presented computer results consider learning rate ($\eta$) =0.1, and gain factor ($\lambda$) =0.5 by referring to two equations (3) & (6) presented in the above.

<table>
<thead>
<tr>
<th>Neurons making a role in learning process</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>learning response time (cycles)</td>
<td>70.65</td>
<td>63.76</td>
<td>16.04</td>
<td>80.80</td>
<td>14.77</td>
<td>7.31</td>
</tr>
</tbody>
</table>

2) Neurons' number effect on learning error

Referring to results for solving reconstruction (pattern recognition) problem solved by a mouse behavior in an 8 figure maze [25][26]. That results measured based on pulsed neuron spikes signals at hippocampus area of a mouse brain. Accordingly, the following table presents observed error values that seem to decrease versus (place field) neuron cells, similar to exponential curve decays to some limit value.

<table>
<thead>
<tr>
<th>No of neuron cells</th>
<th>10</th>
<th>15</th>
<th>18</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean error (cm)</td>
<td>9</td>
<td>6.6</td>
<td>5.4</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Noting that, the value of mean error converges (by increase of number of cells) to some limit, excluded as Cramer-Rao bound. That limiting bound is based on Fisher's information given as tabulated results in the above and derived from [26]. That implies LMS algorithm is valid and obeys the curve shown in blow.

Interestingly, the increase of cooperative neurons' number (at hippocampus of the mouse brain) in learning process is analogous to cooperative activity of ants' number in Ant Colony System (ACS) while solving optimally Travelling Sales-man Problem (TSP). That analogy between ants and neurons behavioral performance is illustrated by some details at two recently published works [27][18].

IV. RESULTS

A. Simulation Results

Referring to the previously published work [28]; that paper deals with analysis and evaluation of learning convergence time using ANN modeling. Therein, it is declared that application of technologically improved educational methodologies implies increasing of learning rate values. That results in better learning performance quality by minimizing of learning convergence (response) time. Therefore, application of presented three teaching methodologies (classical, CAL multimedia modules with visual and with simultaneous auditory and visual tutorial materials). That could be considered as three deferent educational technology levels (representing three teaching methodologies). Consequently, those three methodologies may be mapped (virtually) into three analogous learning rate values. More specifically, the three values($\eta$) =0.05, 0.1, and 0.3 present virtually analogues mapping of the three teaching methodologies respectively : classical, CAL with visual, and CAL with simultaneous auditory and visual materials. Fig.7. Illustrates graphically simulation results of learning performance, considering time response parameter for the three different learning rate values ($\eta$) =0.05, 0.1, and 0.3. The flowchart of suggested simulation computer program is given at APPENDIX C. It is worthy to note that shown three graphs at Fig.7 are in well agreement with obtained case study results given at next subsection 3.2.

A. Case Study Results

A learning style is a relatively stable and consistent set of strategies that an individual prefers to use when engaged in learning [29-30]. Herein, our practical application (case study) adopts one of these strategies namely acquiring learning information through two sensory organs (student eyes and ears). In other words, seen and heard (visual and audible) interactive signals are acquired by student's sensory organs either through his teacher or considering CAL packages (with or without teacher's voice). Practically, children are classified...
in three groups in according to their diverse learning styles (preferences).

After running of computer assessment program for both CAL modules, obtained results are tabulated and graphically presented. The two tables (Table. 5 & Table.6) given in below illustrate obtained practical results after performing three different learning experiments. At table.5, illustrated results are classified in accordance with different students' learning styles following three teaching methodologies. Firstly, the classical learning style is carried out by students-teacher interactive in the classroom. Secondly, learning is taken place using a suggested software learning package without teacher’s voice association. The last experiment is carried out using CAL package that is associated with teacher’s voice. This table gives children's achievements (obtained marks) considering that maximum mark is 100. The statistical analysis of all three experimental marking results is given in details at Table.7 shown in below. Moreover, obtained results are graphically illustrated at two figures (Fig.8 & Fig.9). At Fig.8, graphical comparison of classical learning versus CAL module (without tutor's voice) is presented. However, comparison of classical learning versus CAL module (associated with tutor's voice) is shown at Fig.9.

V. CONCLUSIONS

This piece of research comes to five interesting conclusion remarks presented as follows:

- Evaluation of performance quality for any CAL module is frequently measured after investigational analysis of obtained educational field results. Above presented assessment approach provides educationalists with unbiased fair judgment tool for performance quality measurement of any CAL module based on learning response time.

- Presented two CAL modules with (visual and/or audible) learning materials revealed dependency of learning / teaching effectiveness upon children's sensory cognitive systems. As shown at Table 3, and Fig.6 increasing number of cognitive sensory neurons participating in tutoring processes results in better development of more additive value for educational quality. This remark is well supported by obtained shown results at Table 4, and Fig.7 presenting solving reconstruction (pattern recognition) problem solved by a mouse in 8 figure maze [25-26].

- Detailed comparative assessments for both presented CAL modules versus classical tutoring methodology resulted in superiority of learning outcomes' quality (better learning response time) after application of simultaneous visual and auditory materials (as shown at Fig.8 & Fig.9).

The above two remarks agree well with Lindstrom's findings that participants could only remember 20% of the total learning materials when they were presented with visual material only, 40% when they were presented with both visual and auditory material, and
about 75% when the visual and auditory material were presented simultaneously [7].

- Consequently, by future application of virtual reality technique in learning process will add one more sensory system (tactile) contributing in learning process. So, adding third sensory (tactile system) implies more promising for giving more additive value for learning/teaching effectiveness.

- Finally, for future extension of presented research work, it is highly recommended to consider more elaborate investigational analysis and evaluations for other behavioral learning phenomena observed at educational field (such as learning creativity, improvement of learning performance, learning styles,........etc.) using ANNs modeling. As consequence of all given in above, it is worthy to recommend realistic implementation of ANNs models, to be applicable for solving educational phenomena issues related to cognitive styles observed at educational phenomena and/or activities as that introduced at [31][32][33].

APPENDIX A

Print screen samples for Illustrations of three output phases of designed assessment computer program at Fig.1, Fig.2, and Fig.3.

Fig.1. Basic print screen sample for initial mathematical Long Division process

Fig.2. A print screen for fairly assessment processes results with no mistake.

Fig.3. A print screen for fairly assessment processes results with two mistakes.

APPENDIX B

The shown figure in below illustrates a simplified macro level flowchart describing in brief basic algorithmic steps considered by suggested Computer Assisted Learning package. It is designed in order to perform fairly unbiased assessment process of learning a mathematical topic. After it's running, children time response (scores) are obtained, (samples of print screens is shown at Appendix A). Those samples were obtained in accordance with steps of long division process: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary) as given at reference [14].
APPENDIX C

The shown figure below presents a simplified macro level flowchart describing in brief algorithmic steps for realistic simulation learning program using Artificial Neural Networks. After running that program, three graphs time response results are obtained as shown at Fig.7.

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