

# Variable Structure Control Systems: A Survey

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**Abstract** — The theory of variable structure systems (VSS) with sliding modes is currently one of the most significant research topics within the control engineering domain. Variable structure systems consist of a set of continuous subsystems together with suitable switching logic. Moreover, recently a number of important applications of the theory in the field of power electronics, motion control, robotics, bioprocess etc. have also been reported. Design and analysis for this class of systems are surveyed in this paper. A brief introduction to the theory of sliding mode control, then enumerates the most important novel trends in fundamental research in this field and finally gives some examples of successful engineering applications.

**Keywords** — Variable structure systems, Sliding Mode control, Engineering application.

## I. INTRODUCTION

The theory of variable structure systems (VSS) with sliding modes is currently one of the most significant research topics within the control engineering domain. In addition, the technique provides an easy way to design the control law for a plant, linear or nonlinear. The feasibility of the technique not only has been predicted by theory but also has been demonstrated by numerous computer simulations and hardware experiments.

The idea of sliding mode control (SMC) is to employ different feedback controllers acting on the opposite sides of a predetermined surface in the system state space. Each of those controllers pushes the system representative point (RP) towards the surface, so that the RP approaches the surface, and once it hits the surface for the first time it stays on it ever after. Utkin [1], presented in (1977) a master survey paper on Variable Structure System where he introduced the basic concepts of the VSS, which consists of a set of continuous subsystems together with suitable switching logic. In 1993, the existence of a sliding mode is validated, the control algorithm and data processing used in VSS are analysed. The resulting motion of the system is restricted to the surface, which graphically can be interpreted as “sliding” of the system RP along the surface, [2].

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To demonstrate the Variable Structure System theory and to throw more light on this powerful control law. In the next example a second-order system is considered by the following equation

$$\dot{x} = -\varphi x \quad (1)$$

which possess two structure defined by  $\varphi = \alpha_1^2$  and  $\varphi = \alpha_2^2$ . where  $\alpha_1^2 > \alpha_2^2$ . The phase portrait consists of families of ellipses as given by Figures (1 and 2) which are not asymptotically stable. However, asymptotic stability is achieved if the structure of the system is changed on the co-ordinate axes, that is, if the switching logic is :

$$\Psi = \begin{cases} \alpha_1^2, & \text{if } x\dot{x} > 0 \\ \alpha_2^2, & \text{if } x\dot{x} < 0. \end{cases} \quad (2)$$

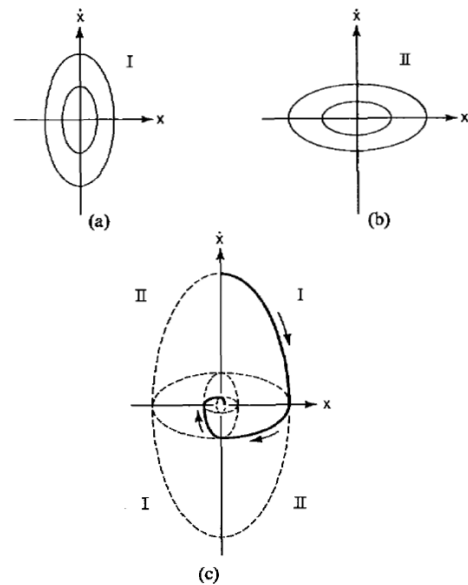


Fig. 1 Asymptotically Stable VSS Consisting of Two Marginally stable Structures

As it has already been mentioned, the switching surface completely determines the plant dynamics in the sliding mode. Therefore, selecting this surface is one of the two major tasks in the process of the SMC system design.

## II. Illustrative Example (Tank Level Control)

Level control is an important aspect in the control of many industrial processes, where the maintenance of a predetermined level of, say, water in a boiler drum, liquids in petrochemical storage tanks, and the level of solution in heat exchangers in food industry is essential for product quality and for the safety of plant and personal.

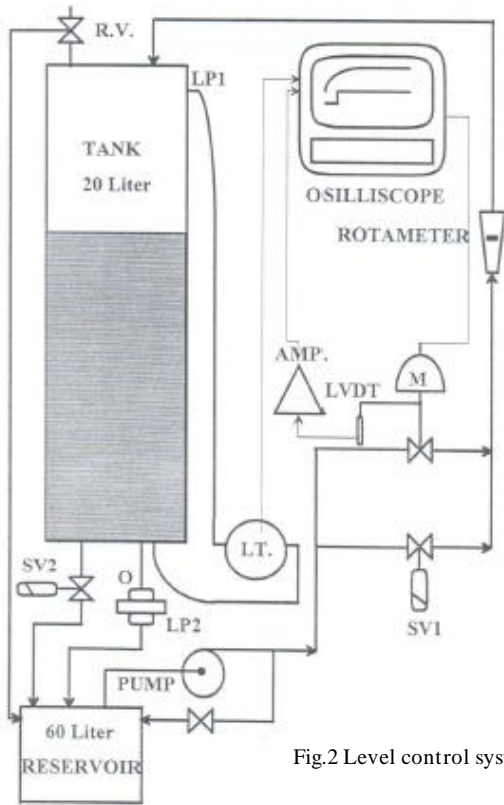


Fig.2 Level control system

In this section the effect and viability of the variable structure controller will be investigated when applied to a drum level control. Figure 1 presents the boiler drum level control loop which is simulated by the laboratory test rig shown in Fig.2. Control performance simulations were conducted and comparisons between VSS method and conventional controllers (PID&LQR) were carried out. Consequently it was concluded that VSS control method is the most powerful control algorithm.

The simulated results are verified by conducting experimental tests to evaluate the dynamics of the feedback level control process upon applying different control strategies including classical optimal PID controller and advanced VSC.

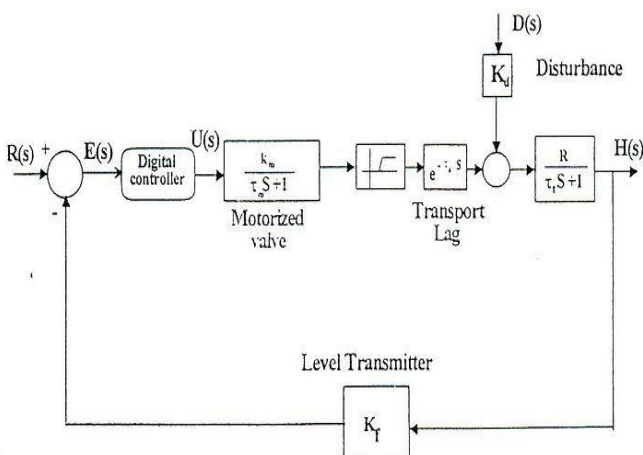


Fig. 3 Level Control System.

The closed-loop process control system is represented by the block diagram shown in Fig. 3

The process is characterized by a transportation delay which can be described by a dead-time. If the dead time  $\tau_d$  is small, then the transportation delay can be approximated by

$$e^{-\tau_d s} \cong \frac{1}{\tau_d s + 1} \quad (3)$$

By comparing the motor time constant 0.740 sec. with tank time constant 48.47 sec. and delay time 3.31 sec., the motor time constant can be neglected. Thus the overall open loop transfer function can be represented by :

$$\frac{H(s)}{E(s)} = \frac{K}{(\tau_r s + 1)(\tau_d s + 1)} \quad (4)$$

In VSS, the controller is allowed to change its structure at any instant from one to another member of a set of possible continuous functions. Design procedure details of the VSS controller were introduced by Utkin [1]. The state equations of the control system are given by :

$$e(t) = r - h(t) * k_j \quad (5)$$

where:  $e(t)$  is error signal,  $r(t)$  is reference head,  $k_j$  is feedback gain and  $h(t)$  is level signal in time domain

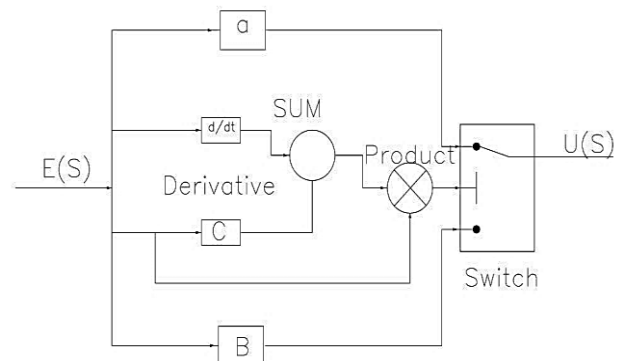
As the sliding plane,  $S = 0$ , is defined by Eq.4 with  $c$  as a positive constant, and the control action  $u$  is defined by Eq.5, then the condition under which the sliding mode exists is given by Eq.6

$$s = ce_1 + e_2$$

$$u = \phi e_1 \quad (6)$$

$$\phi = \begin{cases} \alpha & \text{where } \alpha \geq [-a_1 - c^2 + ca_2] & \text{if } Se_1 > 0 \\ \beta & \text{where } \beta \leq [-a_1 - c^2 + ca_2] & \text{if } Se_1 < 0 \end{cases}$$

The block diagram of the VSS controller is shown in Fig.4.



Figure(3) Level Control System

Fig.4 Variable Structure Controller (VSC).

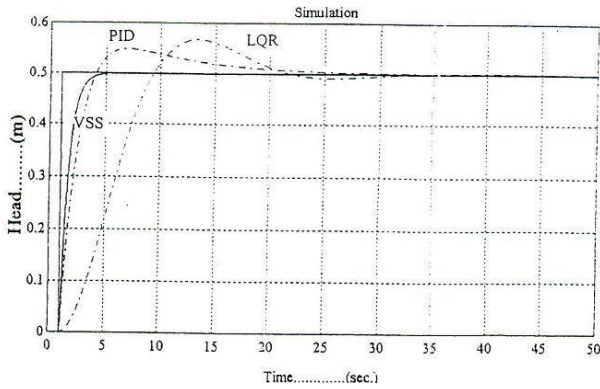


Fig.5 Level Step Responses with LQR, PID and VSS Controller.

In VSS control method the parameter  $c$  of sliding plane is given by  $c = 0.323$ . The gain constants of the control action  $U$  are set as  $a_1 = 15$  and  $a_2 = -11$ . It is confirmed that this combination of parameters gives optimal response, for variation of system parameters from (0 ~ 30%) as shown in Fig.5.

### III. Applications

#### A. Robot Control

The dynamics of an  $n$ -link manipulator are usually modeled by  $n$  coupled second-order nonlinear differential equations. Present-day control methods are often based on nonlinear compensations, which require an accurate manipulator model and, hence, load forecasting. Generally these methods are complex and costly in implementation. Fortunately, the dynamic equations can easily be transformed into a nonlinear canonical form and the matching conditions are always satisfied, so a VSC approach to manipulator control appears to be very promising. Studies using 'two-link manipulators have been reported in [3-7], where a hierarchical VSC law was used. The simulation results showed the existence of chattering, a problem that has been studied in greater detail [2, 6-11].

#### B. Motor Control

Control of electrical motors has been a popular application of VSC. The technique has been applied to the control of dc motors, synchronous motors, and induction motors. The following are just a few of many references in the literature [12-13]. Design of a sliding mode observer for an induction motor has also been done [14].

#### C. Aircraft Control

Variable structure control has been applied to the a variety of flight problems, including (1) control of lateral motion of an aircraft [15], (2) realization of asymptotically decoupled control of roll angle, angle of

attack, and sideslip in the presence of rapid maneuvering [16-21].

#### D. Spacecraft Control

The VSC of multiaxial spacecraft for large-angle rotational maneuvering has been studied to spacecraft rotation damping and orientation control [22-29] has also been reported.

#### E. Other Applications

Listed as follows are some other known applications of

- 1) Load frequency control of power systems [30-33],
- 2) Servomechanisms [34],
- 3) Pulse-width modulation control [35-37],
- 4) Guidance [38-40]
- 5) Process control [41],
- 6) Phase-locked loop control [42-45],
- 7) Power converters [46-47],
- 8) Digital implementation [45, 48],
- 9) Remote vehicle control [49-50],
- 10) Flexible space Structure Control [50-53].

#### NOVEL TRENDS IN SMC

SMC is currently one of the most significant research topics within the control engineering domain. Therefore, in this section we are able to point out only a few, arbitrarily selected issues, which we believe to be the most promising and up to date trends in the field. These include, but by no means are limited to:

- design and implementation of sliding mode observers for systems with unknown inputs [49];
- application of sliding mode observers for fault detection and isolation [54-55];
- higher order sliding modes [2];
- neural network based sliding mode controllers [45];
- sliding mode control of dynamic nonlinear systems by output-feedback [56];
- variable structure control without the reaching phase [57-58];
- sliding mode control application in robotics and complex motion steering systems [59];
- sliding mode control in power electronics and control of electric drives and actuators [60];
- sliding mode control of systems in interaction with their environment [61-62];
- sliding mode control of biotechnological reactors [62].

There are many successful applications of SMC systems reported in literature. Therefore, the examples mentioned above are by no means exhaustive and they may only present a good starting point for further analysis. Most of the examples cited in this section come from the recent special issue of International Journal of Adaptive Control and Signal Processing on Sliding Mode Control, [60].



## VI. CONCLUSIONS

Having traveled a long history of research and development, VSC is well established as a general control method. The paper gives a brief tutorial introduction to the most fundamental issues in the field of variable structure and sliding mode control systems. VSC is naturally attractive to control engineers because its basic concepts are rather easy to understand and has given satisfactory performance in many practical areas of industrial electronics. Moreover, it presents selected novel trends and successful engineering applications in the field.

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