The Effect of Rotation on Thermoelastic Diffusion with Temperature-Dependent Elastic Moduli Comparison of Different Theories

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Abstract—The aim of this paper is to discuss the effect of rotation on thermoelastic diffusion with temperature-dependent elastic moduli. The problem is employed under three theories, the Lord-Shulman and the Green-Lindsay and the classical coupled. By using normal mode method, the expressions for the displacement components, stresses, temperature, concentration and chemical potential are obtained analytically. The numerical results are illustrated graphically in each case considered. A comparison also is made between the three theories in the presence and absence of rotation and the temperature-dependent elastic moduli.

Index Terms—Thermoelasticity, rotation, temperature dependent, diffusion.

I. INTRODUCTION

Earlier, the classical heat conduction equation is used to investigate the thermo-elasticity theory. This theory assumes that the thermal disturbances propagate at infinite speeds "by Hetnarksi [1]". Also, the equation of heat conduction of this theory does not contain elastic terms. The coupled theory of thermoelasticity is introduced "by Biot [2]" to eliminate the second paradox. The theory of thermoelasticity with thermal relaxation times has studied "by Lord and Shulman [3], and Green and Lindsay [4]". They have been developed these theories to eliminate the paradox of infinite speed of the propagation of thermal signals "as discussed by Ignaczak and Starzewski [5]".

A random walk of an ensemble of particles from a region of high concentration to a region of lower concentration is defined as diffusion. The study of these phenomena is due to many applications in geophysics and electronic industry. Diffusion is employed to introduce dopants in controlled amounts into the semiconductor substance integrated circuit fabrication. Also, in Metal Oxide Semiconductor (MOS) transistor, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions. Oil companies use the process of thermo-diffusion to extract oil from oil deposits "as discussed by Kumar and Kansal [6]". Fick's law is used to determine the concentration "as discussed by Mehrer [7]".

The coupled thermoplastic model is used to develop the theory of thermoelastic diffusion "by Nawacki [8-10]". The governing equations for generalized thermo-diffusion in elastic solids have derived "as discussed by Sherief and et al. [11]". On the other hand several authors "as Othman et al. [12], Aouadi [13], Rong-hou et al. [14], Abo-Dahab et al. [15] and
Kumar and Kansal [16] investigated the effect of diffusion on the generalized thermoelasticity for different cases.

Material properties, such as the modulus of elasticity and the thermal conductivity, may be affected by temperature dependent. The temperature dependence of material properties is neglected when the temperature variation from the initial temperature is low. The temperature dependence of material properties is considered when the temperature changes very high. The reactor vessels, turbine engines, space vehicles and refractory industries are affected by high temperature changes. If the temperature dependence of material properties is neglected, this is due to significant errors "as discussed by Noda [17]". Extensive studies "by Ezzat et al. [18]; Aouadi [19]; Othman and Song [20]; Othman et al. [21]; Othman [22]" have been discussed the temperature dependence of material properties.

The thermoelastic plane waves without energy dissipation in a rotating body have studied "by Chanderashekariah and Srinath [23]". The effect of rotation on plane waves in generalized thermoelasticity with two relaxation times has investigated "by Othman [24]". The effect of rotation on Rayleigh-Lamb waves in an isotropic generalized thermoelastic diffusion plate is studied "by Kumar and Kansal [25]".

In the present paper, we will study the effect of rotation on thermoelastic diffusion with temperature-dependent elastic moduli comparison of different theories.

II. FORMULATION OF THE PROBLEM

We consider rotation of a homogeneous isotropic generalized thermodiffusive medium. We will use a rectangular coordinate system \((x, y, z)\) and \(z\)-axis is directed vertically downward. The elastic medium is rotating with angular velocity \(\Omega = (0, \Omega, 0)\), we assume that the displacement vector has two components \((u, 0, w)\), the temperature \(T(x, z, t)\) and the concentration \(C(x, z, t)\).

Following "Kumar [26]" the governing equations for an isotropic homogeneous elastic solid with generalized thermoelastic diffusion in the absence of the body forces are:

1. The constitutive relations
   \[
   \sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \nu (\theta + \tau_1(\dot{\theta}))) - \beta(C + \tau^1\dot{C})
   \]

2. The equation of motion
   \[
   \rho \frac{d^2 u_{i,j}}{dt^2} + (\lambda + \mu) u_{i,j} - \nu (\theta + \tau_1(\dot{\theta}))_{,i} - \beta(C + \tau^1\dot{C})_{,i} = \rho[\{0, \{\Omega \wedge (\Omega \wedge u)\}_{,i} + (2\Omega \wedge u)_{,i}\}
   \]

3. The equation of heat conduction
   \[
   \rho C_e (\theta + \tau_0(\dot{\theta})) + \nu T_0 e_{kk} + \varkappa \tau_0(\dot{\theta}) + \psi \tau_0(\dot{\theta}) + aT_0(C + \gamma \dot{C}) = K\theta_{,ii}
   \]

4. The equation of mass diffusion
   \[
   d\beta e_{kk,ii} + d\alpha (\theta + \tau_0(\dot{\theta}))_{,ii} + (C + \psi \theta^2) - d\beta(C + \gamma \dot{\theta})_{,ii} = 0
   \]

5. The strain displacement relation is given by
   \[
   e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})
   \]

In the above equation, the symbols "," and "." denote the material derivative and the derivative w.r.t t time and \(i, j = x, y, z, \rho, C, a, b\) are the density, the specific heat at constant strain, the measure of thermoelastic diffusion effects and diffusion effects, respectively. \(\theta = T - T_0\), \(T_0\) is the temperature of the medium in its natural state assumed to be such that \(|\theta|/T_0| < < 1\). \(\tau^0\), \(\tau^1\), \(\zeta\) and \(\gamma\) are the diffusion relaxation times and \(\tau_0\), \(\tau_1\) are thermal relaxation times. \(S\) and \(P\) are the entropy per unit mass and the chemical potential, respectively, \(\lambda, \mu\) are Lamé’s constants. \(\alpha = (3\lambda + 2\mu)\alpha_1\), \(\alpha_1\) is the coefficient of linear thermal expansion and \(\beta = (3\lambda + 2\mu)\alpha_2\), \(\alpha_2\) is the coefficient of linear diffusion expansion, \(d\) and \(K\) are the thermoelastic diffusion constant and the thermal conductivity respectively.

We will study the above equations for the following three different theories:

(i) Classical and Dynamical coupled theory (CD)
s = \eta = \zeta = \gamma = e = \tau_0 = \tau' = \tau = 0.
(ii) Lord and Shulman's theory (LS)
\eta = \zeta = \tau = 0, e = 1, \gamma = \tau_0.
(iii) Green and Lindsay's theory (GL)
\eta = \tau_0, \zeta = \tau = 0, e = 0, \gamma = \tau = 0.

We assume that
\lambda = \lambda_0(1-\alpha^*T_0), \mu = \mu_0(1-\alpha^*T_0), \nu = \nu_0(1-\alpha^*T_0),
\beta = \beta_0(1-\alpha^*T_0).
Where \lambda_0, \mu_0, \nu_0, and \beta_0 are constants and \alpha^* is the empirical constant. In the case of the modulus of elasticity are independent of temperature \alpha^* = 0.

Using "equations (7) and (11) in equation (1)," we obtain
\[ \alpha\sigma_{xx} = \frac{1}{\alpha}(2\mu_0 + \lambda_0) \frac{\partial u}{\partial x} + \lambda_0 \frac{\partial w}{\partial z} - v_0(\theta + \tau_0) - \beta_0(C + \tau^2C), \]
\[ \alpha\sigma_{zz} = \frac{1}{\alpha}(2\mu_0 + \lambda_0) \frac{\partial w}{\partial z} + \lambda_0 \frac{\partial u}{\partial x} - v_0(\theta + \tau_0) - \beta_0(C + \tau^2C), \]
\[ \alpha\sigma_{xz} = \mu_0 \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right), \]
where \alpha = 1/(1-\alpha^*T_0), \nu = (3\lambda_0 + 2\mu_0)\alpha_1, and
\[ \beta_0 = (3\lambda_0 + 2\mu_0)\alpha_\epsilon. \]

Substituting from "equation (11) in equation (4)," we get
\[ \rho[\ddot{u} - \Omega^2 u + 2\Omega \dot{w}] = \frac{1}{\alpha}(\mu_0 + \lambda_0) \frac{\partial e}{\partial x} + \mu_0\nabla^2 u - v_0(\theta + \tau_0) - \beta_0(C + \tau^2C), \]
\[ \rho[\ddot{w} - \Omega^2 w + 2\Omega \dot{u}] = \frac{1}{\alpha}(\mu_0 + \lambda_0) \frac{\partial e}{\partial z} - v_0(\theta + \tau_0) - \beta_0(C + \tau^2C). \]

Using "equations (7) and (11) in equations (3), (5) and (6)," we obtain
\[ K\theta_{ii} = \rho C_E (\theta + \tau_0) + \frac{\nu T_0}{\alpha} \left( \varepsilon_{kk} + \tau_0 \varepsilon_{kk} \right) + a T_0 (C + \gamma C). \]
\[ \frac{d\beta_0}{\alpha} e_{kk,ii} + \lambda_0(\theta + \tau_0)_{,ii} + (C + \gamma C)_{,ii} - db(C + \tau^2C)_{,ii} = 0, \]
\[ P = -\frac{\beta_0}{\alpha} e_{kk} + b(C + \tau^2C) - a(\theta + \tau_0). \]
It is convenient to introduce the dimensionless variables as
\[ (x', z') = \frac{\theta}{c_1}(x, z), \quad (u', w') = \frac{\theta}{c_1}(u, w), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\rho c_1^2}, \]
\[ \theta' = \frac{v_0\theta}{\rho c_1^2}, \quad (r', r', r', r') = \delta_0(r^0, r', r_0, r_1), \quad P' = \frac{P}{\beta_0}. \]

Using "equations (7)-(9) in the dimensionless form, where we have dropped the prime for convenience, take the form
\[ [\ddot{u} - \Omega^2 u + 2\Omega \dot{w}] = \frac{1}{\alpha}[\beta_1(\theta + \tau_0) + (1 - \beta_1)\nabla^2 u - \frac{\partial e}{\partial x} + (\theta + \tau_0) \frac{\partial e}{\partial z} - \beta_0(C + \tau^2C)], \]
\[ [\ddot{w} - \Omega^2 w + 2\Omega \dot{u}] = \frac{1}{\alpha}[\beta_1(\theta + \tau_0) + (1 - \beta_1)\nabla^2 w + \frac{\partial e}{\partial z} - (\theta + \tau_0) \frac{\partial e}{\partial x} - \beta_0(C + \tau^2C)]. \]
\[ \nabla^2 \theta = (\theta + \tau_0)\theta + \frac{\delta_1}{\alpha} (\dot{e} + \varepsilon \dot{e}) + \delta_2 \delta_0 (C + \gamma \dot{C}), \]
\[ \nabla^2 e + \alpha_2 (\nabla^2 \theta + \tau_1 \nabla^2 \dot{\theta}) + \alpha_3 (C + \varepsilon \dot{e}) - \alpha_4 (\nabla^2 C + \tau_1 \nabla^2 \dot{C}) = 0, \]
\[ \alpha P = -e + \alpha_4 (C + \tau^2C) - \alpha_2 (\theta + \tau_0), \]
\[ \alpha \sigma_{xx} = \frac{\partial u}{\partial x} + (2\beta_1 - 1) \frac{\partial w}{\partial z} - (\theta + \tau_0) - (C + \tau^2C), \]
\[ \alpha \sigma_{zz} = \frac{\partial w}{\partial z} + (2\beta_1 - 1) \frac{\partial u}{\partial x} - (\theta + \tau_0) - (C + \tau^2C), \]
\[ \alpha \sigma_{xz} = (1 - \beta_1) \frac{\partial w}{\partial x} + (1 - \beta_1) \frac{\partial u}{\partial z}. \]
Where \delta_1 = \frac{\nu_0}{\rho C_E}, \delta_2 = \frac{a c_1^2}{C_E \beta_0}, \alpha_2 = \frac{\rho c_1^2 a \alpha}{\nu_0 \beta_0}, \alpha_3 = \frac{K c_1^2 \alpha}{C_E d \beta_0}, \alpha_4 = \frac{\alpha_3 c_1^2 b \beta_0}{\beta_0}, \beta_i = \frac{\mu_0 + \lambda_0}{\rho c_1^2}. 
By introducing the displacement potentials \phi and \psi, we can write
\[ u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \]
Substitution of "equation (30) in equations (21)- (25)," leads to
\[
[D^0 - A_1D^8 + A_2D^6 - A_3D^4 + A_4D^2 - A_5]\phi^* = 0 .
\]
where \( A_1 = \frac{s_1}{s_0} \), \( A_2 = \frac{s_2}{s_0} \), \( A_3 = \frac{s_3}{s_0} \), \( A_4 = \frac{s_4}{s_0} \), \( A_5 = \frac{s_5}{s_0} \) and \( s_0, s_1, s_2, s_3, s_4 \) and \( s_5 \) are given in the Appendix.

Similarly, we can get
\[
[D^{10} - A_1D^8 + A_2D^6 - A_3D^4 + A_4D^2] [\psi^*, \theta^*, C^*] = 0,
\]
We can factorized "equation (43)," in the form

\[
(D^2 - K_1^2)(D^2 - K_2^2)(D^2 - K_3^2)\phi^* = 0 .
\]
Where \( k_i^2 \) are the roots of the characteristic equation of "equation (43)."

The solution of "equation (45)," must be bounded as \( z \rightarrow \infty \). Then,

\[
D^0 \phi = \sum_{i=1}^{5} G_i e^{-K_i z},
\]

\[
\psi^* = \sum_{i=1}^{5} R_i G_i e^{-K_i z},
\]

\[
\theta^* = \sum_{i=1}^{5} Q_i G_i e^{-K_i z},
\]

\[
C^* = \sum_{i=1}^{5} M_i G_i e^{-K_i z}.
\]
where \( R_i, Q_i \) and \( M_i \) are given in the Appendix.

Using "equations (46) and (47) into equation (30)," we obtain

\[
\psi = \sum_{i=1}^{5} (is + K_i R_i) G_i e^{-K_i z},
\]

\[
w^* = \sum_{i=1}^{5} (-K_i + isR_i) G_i e^{-K_i z}.
\]

Substituting "equations (48)-(51) in equations (25)- (28)," yields

\[
P^* = \sum_{i=1}^{5} H_i G_i e^{-K_i z},
\]

\[
\sigma_{xx}^* = \sum_{i=1}^{5} L_i G_i e^{-K_i z},
\]

\[
\sigma_{zz}^* = \sum_{i=1}^{5} J_i G_i e^{-K_i z},
\]

\[
\sigma_{xz}^* = \sum_{i=1}^{5} F_i G_i e^{-K_i z}.
\]
Where \( H_i, L_i, J_i \) and \( F_i \) are given in the Appendix.

Using "equation (37) in the boundary conditions (36)," we get
\[ \sigma_{xz}^* = 0, \quad \sigma_{zz}^* = -F_0, \quad \frac{\partial \theta^*}{\partial z} = 0, \quad \frac{\partial C^*}{\partial z} = 0, \quad \theta^* = n_0 \] at \( z = 0 \). \quad (56)

Substituting "equations (48), (49), (54), (55) in the boundary conditions (56)," we get

\[ \sum_{i=1}^{5} F G_i = 0, \quad (57) \]
\[ \sum_{i=1}^{5} J_i G_i = -F_0, \quad (58) \]
\[ \sum_{i=1}^{5} K_i Q_i G_i = 0, \quad (59) \]
\[ \sum_{i=1}^{5} K_i M_i G_i = 0, \quad (60) \]
\[ \sum_{i=1}^{5} Q_i G_i = n_0. \quad (61) \]

The MATLAB program is used to obtain the constants \( G_i, \quad i = 1, 2, 3, 4, 5 \).

IV. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the theoretical results in the preceding section, we take the physical constants as \( K = 386 \text{ (J/KgK)}, \quad T_0 = 293K, \quad t = 0.1 \text{ s}, \quad \tau^0 = 0.2 \text{ s}, \quad F_0 = 1.5, \quad \tau_0 = 0.1 \text{ s}, \quad \rho = 8954 \text{ (Kg/m}^3), \quad C_E = 383.1, \quad \alpha_i = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad \alpha_c = 1.98 \times 10^{-4} \text{ (m}^3/\text{Kg}), \quad \lambda_0 = 7.76 \times 10^{10} \text{ Kg/(ms}^2), \quad \mu_0 = 3.86 \times 10^{10} \text{ Kg/(ms}^2), \quad \alpha = 0.0012, \quad n_0 = 1, \quad \tau_1 = 0.3 \text{ s}, \quad \tau^1 = 0.4 \text{ s}, \quad d = 0.85 \times 10^{-8} \text{ Kgs/m}^3, \quad a = 1.2 \times 10^3 \text{ m}^2/(s}^2 \text{K)}, \quad b = 0.9 \times 10^6 \text{ m}^5 /\text{(Kgs}^2), \quad x = 1, \quad s = 1.

Since \( \omega \) is complex, then we take \( \omega = \omega_0 + i \zeta, \quad \omega_0 = -2, \quad \zeta = 1 \).

By using the numerical technique, the distribution of temperature \( \theta \), the displacement components \( u, w \), the stress components \( \sigma_{xz}, \sigma_{zz} \), and the concentration \( C \) are given in Figures (1)–(6). In these figures, there exist two groups of curves. The first one represents the three different theories of thermoelasticity without rotation i.e. \( \Omega = 0 \) and the second represents the same theories of thermo-elasticity with rotation i.e. \( \Omega = 0.3 \). In these Figures, the solid lines, dashed lines and small dashed lines corresponds for CD, L-S and G-L theories respectively.

Figure 1 represents the variation of the displacement component \( u \) versus \( z \). It starts from positive values in the absence of rotation but it starts from negative values in the presence of rotation. For three theories without rotation, the values of displacement component \( u \) decrease to a minimum value in the range \( 0 \leq z \leq 1.06 \). Also, it can be seen that the values of \( u \) increase to a maximum value in the range \( 1.06 \leq z \leq 2.69 \) for CD, L-S and G-L theories respectively, and move in wave propagation. In the presence of rotation, the values of the displacement component \( u \) increase in the range \( 0 \leq z \leq 0.19 \) for CD and L-S theories and in the range \( 0 \leq z \leq 0.2 \) for G-L theory. Also, it can be seen that \( u \) decrease in the range \( 0.19 \leq z \leq 0.8 \) and increase to a maximum value in the range \( 0.8 \leq z \leq 3.28 \) for CD and L-S theories.

Figure 2 displays the variation of displacement component \( w \) versus \( z \). In the presence of rotation, the displacement component \( w \) begins from the negative value for G-L theory. For CD and L-S theories, but it begins from the negative value for G-L theory. For CD and L-S theories, the displacement component \( w \) decreases in the range \( 0 \leq z \leq 4 \), but for G-L theory, it increases in the range \( 0.2 \leq z \leq 3.63 \) and then with the increase in \( z \) values converges towards zero value. In the absence of rotation, for the three theories, the displacement component \( w \) begins from negative values. For the CD and L-S theories, the values of the displacement component \( w \) increase in the range \( 0 \leq z \leq 0.46 \), but they decrease in the range \( 0.35 \leq z \leq 0.9 \) and then increase in the range \( 0.9 \leq z \leq 2.69 \). For the G-L theory, the values of the displacement component \( w \) decrease in the range \( 0 \leq z \leq 0.6 \), then increase in the range \( 0.6 \leq z \leq 2.33 \) and move in a wave propagation.
Figure 3 shows that the variation of the concentration $C$ at $\Omega = 0$ and $\Omega = 0.3$. It is noticed that the values of concentration $C$ at $\Omega = 0$ starts from positive values but start from negative values at $\Omega = 0.3$. For CD and L-S theories at $\Omega = 0$, the values of concentration $C$ decrease to a minimum value in the range $0 \leq z \leq 0.67$, then enlarge to a maximum value in the range $0.67 \leq z \leq 2.22$ and move in a wave propagation. For G-L theory, the values of concentration $C$ decrease to a minimum value in the range $0 \leq z \leq 0.96$, then enlarge to a maximum value in the range $0.96 \leq z \leq 2.85$ and move as a wave. In case of $\Omega = 0.3$, the values of concentration $C$ increase in the range $0 \leq z \leq 3.42$ for three theories and then with increase in $z$, values converges towards zero.

As clear from Figure 4 which is a plot of the variation of temperature $\theta$ with $z$, which indicate that values of $\theta$ at $\Omega = 0$ decrease to minimum values in the ranges $0 \leq z \leq 1.44$, $0 \leq z \leq 1.67$ and $0 \leq z \leq 1.69$ for CD, L-S and G-L theories respectively. Also, it can be seen that the values of temperature $\theta$ at $\Omega = 0$ increase to maximum values in the ranges $1.44 \leq z \leq 3.06$, $1.67 \leq z \leq 3.4$ and $1.69 \leq z \leq 3.92$ for CD, L-S and G-L theories and move as a wave. It is noticed that the values of temperature $\theta$ at $\Omega = 0.3$ decrease from maximum values to minimum values in the range $0 \leq z \leq 3.03$ for three theories and then with increase in $z$ values converges towards zero.

The variation for $\sigma_{xz}$ with $z$ is shown in Figure 5. It is evident that the values of $\sigma_{xz}$ at $\Omega = 0.3$ decrease to a minimum value in the range $0 \leq z \leq 0.23$, then enlarge to a maximum value in the range $0.23 \leq z \leq 1.9$ of the three theories and then with increase in $z$ values converges towards zero. It can be seen that the values of $\sigma_{xz}$ at $\Omega = 0$ decrease in the range $0 \leq z \leq 0.5$ for all theories. Also, it is observed that the values of $\sigma_{xz}$ enlarging to maximum values in the ranges $0.5 \leq z \leq 1.76$, $0.5 \leq z \leq 1.67$, $0.5 \leq z \leq 1.9$ for CD, L-S and G-L theories respectively and move in a wave propagation.

Figure 6 displays the variation of $\sigma_{zz}$ with $z$. It is noticed that the values of $\sigma_{zz}$ at $\Omega = 0.3$ increase with greater magnitude in the range $0 \leq z \leq 0.67$, then with the increase in $z$ values converge towards zero value. It can be seen that the values of $\sigma_{zz}$ at $\Omega = 0$ enlarge to maximum values in the ranges $0 \leq z \leq 0.9$, $0 \leq z \leq 0.98$ and $0 \leq z \leq 0.76$ for CD, L-S and G-L theories respectively. Also, it is seen that the values of $\sigma_{zz}$ at $\Omega = 0$ decrease to minimum values in the ranges $0.9 \leq z \leq 2.38$, $0.98 \leq z \leq 2.36$ and $0.76 \leq z \leq 2.1$ for CD, L-S and G-L theories respectively and move in wave propagation.

Figures 7-12 represent the distribution of the displacement components $u, w$, the temperature $\theta$, the concentration $C$ and the stress components $\sigma_{xz}$ and $\sigma_{zz}$ in the case of a material with temperature dependent and temperature independent. In these figures, the solid lines, dashed lines and small dashed lines corresponds for the CD, L-S and G-L theories respectively.

Figure 7 displays the variation of the displacement component $u$ versus $z$. In case of the dependent temperature, the values of the displacement component $u$ start from negative values. For CD and L-S theories, the values of the displacement $u$ increase in the range $0 \leq z \leq 0.193$, but they decrease in the range $0.193 \leq z \leq 0.735$ and then increase in the range $0.735 \leq z \leq 3.6$. For the G-L theory, the values of the displacement $u$ increase in the range $0 \leq z \leq 0.33$, then decrease in the range $0.33 \leq z \leq 0.62$, then increase in the range $0.62 \leq z \leq 2.81$ and move in wave propagation. In case of the independent temperature, the values of the displacement component $u$ begin from negative values. For CD and L-S theories, they increase in the range $0 \leq z \leq 0.261$, but they decrease in the range $0.261 \leq z \leq 1.03$ then increase in the range $1.03 \leq z \leq 3.69$. In G-L theory, the values of the displacement component $u$ increase in the
The curves in the context of the CD, L-S and G-L theories decrease exponentially with increasing $\zeta$, this indicates that the thermoelastic waves are unwanted and non-dispersive, where purely thermoelastic waves undergo both attenuation and dispersion.

2. The diffusion phenomenon is discussed; therefore it is used to improve the conditions of oil extraction. These theories are employed to investigate various types of geophysical and industrial applications.

3. The value of all the physical quantities converges to zero with increasing the distance $\zeta$.

4. It's obvious the rotation and the temperature-dependent elastic moduli have an important role in the distribution of the field quantities.

5. The method which used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity.

### APPENDIX

\[
r_1 = s^2 + \alpha(\alpha^2 - \Omega^2), \quad r_2 = s^2 + \frac{\alpha}{(1 - \beta_1)}(\alpha^2 - \Omega^2),
\]
\[
r_3 = \frac{2\alpha \Omega \omega}{(1 - \beta_1)},
\]
\[
r_4 = s^2 + \alpha(1 + \beta_1 \omega), \quad r_5 = \frac{\delta_1 \delta_0}{\alpha} \omega(1 + \varepsilon_1 \omega),
\]
\[
r_6 = \delta_0 \delta_0 \omega(1 + \gamma_1 \omega), \quad r_7 = \alpha_2(1 + \beta_1 \omega), \quad r_8 = \alpha_4(1 + \gamma_1 \omega),
\]
\[
r_9 = \alpha_4(1 + \varepsilon_1 \omega) \omega, \quad s_0 = \alpha_4(1 + \gamma_1 \omega)(\alpha_4 - 1),
\]
\[
s_1 = r_8(2r_4 + s^2 + r_2 + r_1) + r_6[s^2 r_4 + (r_2 + r_1 + r_4)(r_4 + s^2) + r_4 r_2 + r_4(r_2 + r_1) + 2\alpha \Omega r_1]
\]
\[
+ \tau_0[2r_4 + r_2 + r_1] + (1 + \tau_0 \omega)(r_5[r_4(s^2 + r_5)] + r_5[r_4 + 2s^2 + r_5]) + (1 + \gamma_1 \omega)(r_7(r_4 - r_5))]
\]
\[
+ (1 + \gamma_1 \omega)(-s^4 + 2s^2(r_5 - r_4) + (r_4 + r_2)[r_4(r_5 - r_4) - 2r_4])
\]
\[
s_1 = (1 + \tau_0 \omega)((r_4 + r_2)(r_4(s^2 + r_5) + r_4 s^2 + r_5(s^2 + r_5)) + r_4 r_2 + r_4(s^2 + r_5) + r_5 s^2 + r_5)
\]
\[
+ r_4 r_2 + r_4(r_2 + r_1) + (1 + \tau_0 \omega)((r_4 - r_5)[s^4 + 2(r_4 + r_2)s^2 + r_4] - s^2(r_4 + r_2) + 2r_4)] + (1 + \tau_0 \omega)[(r_4 - r_5)][s^3
\]
\[
+ \tau_0(r_4 + 2\alpha \Omega r_1)]
\]
\[ s_4 = (1 + \tau_1 \omega) s_3 \{ (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) + t_7 (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \} \]

\[ - t_5 s_3 \{ (t_6 - t_4) (s_3 + 2t_5) - \tau_3 s_3 \} + (1 + \tau_1 \omega) s_3 \{ (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) + t_7 (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \} \]

\[ - t_5 s_3 \{ (t_6 - t_4) (s_3 + 2t_5) - \tau_3 s_3 \} + (1 + \tau_1 \omega) s_3 \{ (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) + t_7 (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \} \]

\[ s_5 = (1 + \tau_1 \omega) r_2 s_3 \{ (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) + t_7 (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \} \]

\[ + (1 + \tau_1 \omega) r_2 s_3 \{ (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) + t_7 (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \} \]

\[ + r_2 (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \]

\[ + r_2 (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5) \]

\[ R_i = - \frac{r_i}{(K_i^3 - \tau_i)} \]

\[ Q_i = \frac{O_i}{N_i} \]

\[ M_i = \frac{E_i}{N_i} \]

\[ Q_i = [r_2 (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5)] \]

\[ N_i = [r_2 (t_5 + t_6 - \tau_4 \omega) (s_3 + 2t_5) q t_0 + s_4 (t_6 + t_7) (\tau_4 \omega + s_3 + 2t_5)] \]

\[ F_i = \left( \frac{\beta_i - 1}{\alpha} \right) \{ 2 i s + (s^2 + K_i^3) R_i \} \]

\[ E_i = \left( \frac{\beta_i - 1}{\alpha} \right) \{ 2 i s + (s^2 + K_i^3) R_i \} \]

\[ H_i = \frac{1}{\alpha} \left( s^2 - K_i^3 + \alpha_4 (1 + \tau_1 \omega) M_i - \alpha_2 (1 + \tau_1 \omega) Q_i \right) \]

\[ L_i = \frac{1}{\alpha} \left[ (1 + \tau_1 \omega) Q_i - (1 + \tau_1 \omega) M_i \right] \]

\[ J_i = \frac{1}{\alpha} \left[ (1 + \tau_1 \omega) Q_i - (1 + \tau_1 \omega) M_i \right] \]

REFERENCES


Fig. 1 Variation of the displacement component \( u \) versus \( z \)
Fig. 2 Variation of the displacement component w versus z

Fig. 3 Variation of the concentration C versus z

Fig. 4 Variation of the temperature θ versus z

Fig. 5 Variation of the stress σxz versus z

Fig. 6 Variation of the stress σzz versus z

Fig. 7 Variation of the displacement component u versus z

Fig. 8 Variation of the displacement component w versus z

Fig. 9 Variation of the temperature θ versus z

Fig. 10 Variation of the concentration C versus z

Fig. 11 Variation of the stress σxz versus z
Fig. 12 Variation of the stress $\sigma_{zz}$ versus $z$

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