Abstract — The present investigation is concerned with effect of rotation on the wave propagation of generalized thermoelastic diffusion with one relaxation time. The exact expressions for temperature, the displacement components, stress components, chemical potential and concentration are obtained and represented graphically. Numerical computations are carried out and are obtained by using the normal mode analysis. Comparisons are made for different values of relaxation time in the presence and absence of rotation.

Keywords — Generalized thermoelasticity, thermoelastic diffusion, L-S theory, rotation.

I. INTRODUCTION

The propagation of waves in thermoelastic materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipe and metallurgy. The importance of thermal stresses in causing structural damages and changes in functioning of structure is well recognized whenever thermal stress environments are involved. Therefore, the ability to predict electrodynamics stress induced by sudden thermal loading in composite structures is essential for the proper and safe design and the knowledge of its response during the last three decades, non-classical theories of thermoelastic so-called generalized thermoelastic.

The coupled theory of thermoelastic to deal with a defect of the uncoupled theory that mechanical causes have no effect on the temperature was developed "by Biot [1]". However, this theory shares a defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves. The theory of generalized thermo-elasticity with one relaxation time for the special case of an isotropic body was introduced "by Lord and Shulman [2]". This theory was extended "by Sherief [3] and Dhaliwal and Sherief [4]" to include the anisotropic case. In this theory, a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and coupled theories of thermoelasticity. For this theory, "Ignaczak [5]" studied uniqueness of solution; "Sherief [6]" proved uniqueness and stability. L-S theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity was used "by Othman [7]". The effect of rotation on plane waves in generalized thermoelasticity with two relaxation times was studied "by Othman [8]".

The thermo diffusion in elastic solids is due to coupling of the fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with environment. The theory of thermoelastic diffusion by using coupled thermoelastic model was developed "by Nowacki [9, 10]".

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Diffusion can be defined as the random walk, an ensemble of particles, from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "dopants" in controlled amount to semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source drain regions in MOS transistors and dope ploy–silicon gates in MOS transistors. In most of these applications, the concentration is calculated using what is known as Ficks law. This is a simple law that does not take in to consideration the mutual interaction between the introduced substance and the medium in to which it is introduced or the effect of the temperature on this interaction. The theory of thermoelastic diffusion was developed "by Nowacki [11, 12]”. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Recently, the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves was developed "by Sherief et al. [13]". The effect of magnetic field on plane waves in rotating media in thermoelasticity of type II (G-N model) was studied "by Roychoudhuri et al. [14]". "Sherief and Saleh [15]” worked on a one dimensional problem of a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. The effect of rotation on elastic waves both partial and surface acoustic waves in a piezoelectric half-space. The modal of two dimensional equations of generalized thermoelasticity with one and two relaxation times under the effect of rotation was established "by Othman [16]”. The dependence of the modulus of elasticity on reference temperature in the theory of generalized thermoelastic diffusion with one relaxation time was studied "by Othman et al. [17]”. The normal mode analysis was applied "by Othman et al. [18]” to study the effect of magnetic field and thermal relaxation on 2-D problem of generalized thermoelastic diffusion. The effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation was studied "by Othman and Song [19]” . The effect of fractional parameter on plane waves of generalized magneto-thermo-elastic diffusion with temperature dependent elastic medium was investigated "by Othman et al. [20]". The effect of rotation on a thermoelastic diffusion with temperature-dependent elastic moduli comparison of different theories was studied "by Elmaklizi and Othman [21]”.

In this paper, the coupled and Lord-Shulman theories are applied on a problem of a thermoelastic diffusion half-space. The normal mode analysis is used to obtain the exact expressions for the temperature, displacement components, normal stress components, chemical potential and concentration. The distribution of the temperature, displacement, normal stress component and concentration are represented graphically for different values of relaxation time in the presence and absence of rotation.

II. FORMULATION OF THE PROBLEM
Following, "Sherief et al. [13] and Roychoudhuri et al. [14]" the governing equations for an isotropic, homogenous elastic solid with generalized thermodiffusion at constant temperature $T_0$ in the undisturbed state, in the absent of body forces and heat load are

\begin{equation}
\rho \left[ \ddot{u}_i + \left\{ \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) \right\}_i + (2\mathbf{\Omega} \times \dot{\mathbf{u}})_i \right] = \mu \mathbf{u}_{ij,j} + (\lambda + \mu) \mathbf{u}_{ij,j} - \beta_1 T_{,i} - \beta_2 C_{,i},
\end{equation}

where $u_i$ are components of the displacement vector, $\mathbf{\Omega} = \mathbf{\Omega} \mathbf{n}$ is an angular velocity, where $\mathbf{n}$ is the unit vector representing the direction of the axis of rotation, $T$ is the absolute temperature, $C$ is the concentration of diffusive material in the elastic, $\lambda, \mu \, \text{are lame's constants, } \rho \, \text{is the density and } \beta_1, \beta_2 \, \text{are material constants given by} \beta_1 = (3\lambda + 2\mu)\alpha, \, \alpha \, \text{is the coefficient of linear thermal}$
expansion, \( \beta_2 = (3\lambda + 2\mu)\alpha_c \) and \( \alpha_c \) is the coefficient of linear diffusion expansion. 

ii) The equation of energy

\[
K_{T,ii} = \left[ \frac{\partial^2}{\partial t^2} + \tau_0 \frac{\partial^2}{\partial t^2} \right] \left[ \rho C_E T + \beta_1 T_0 e_{kk} + a T_0 C \right].
\]

(2)

where \( K \) is the thermal conductivity, \( \tau_0 \) is the thermal relaxation time, \( C_E \) is the specific heat at constant strain, \( a \) is a measure of thermodiffusion effect, \( T_0 \) is a reference temperature assumed to obey the inequality \( \left| (T - T_0)/T_0 \right| < 1 \) and \( e_{ij} \) are the components of the strain tensor given by

\[
e_{ij} = \frac{1}{2} \left( u_{ij} + u_{ji} \right).
\]

(3)

(iii) The equation of diffusion has the form

\[
d\beta_2 e_{kk,ii} + d a T_{,ii} + \dot{C} + \tau \dot{C} - db_{,ii} = 0.
\]

(4)

where \( d \) is the diffusion coefficient, \( b \) is a measure diffusion effect and \( \tau \) is the diffusion relaxation time.

The constitutive equations have the form

\[
\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} \left[ \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C \right].
\]

(5)

\[
P = -\beta_2 e_{kk} + b C - a (T - T_0).
\]

(6)

Where, \( \sigma_{ij} \) is the components of stress tensor and \( P \) is the chemical potential.

From Eq. (5), it follow that the stress tensor components have the form

\[
\sigma_{xx} = 2\mu e_{xx} + \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C,
\]

(7)

\[
\sigma_{zz} = 2\mu e_{zz} + \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C,
\]

(8)

\[
\sigma_{xz} = 2\mu e_{xz},
\]

(9)

\[
\sigma_{yy} = \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C,
\]

(10)

\[
\sigma_{xy} = \sigma_{yx} = 0.
\]

(11)

It follow from the description of the problem that all considered functions will depend on \( x, z \) and \( t \). We thus obtain the displacement components of the form \( \vec{u}=(u_1, u_2, u_3) \) also assume that the plate is rotating about an axis normal to the plate \( \Omega = (0, \Omega, 0) \). Eq. (1) can be rewritten as

\[
\rho \left[ \ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3 \right] = \mu \nabla^2 \dot{u}_1 + (\lambda + \mu) \frac{\partial e}{\partial x} - \beta_1 \frac{\partial T}{\partial x} - \beta_2 \frac{\partial C}{\partial x},
\]

(12)

\[
\rho \left[ \ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1 \right] = \mu \nabla^2 \dot{u}_3 + (\lambda + \mu) \frac{\partial e}{\partial z} - \beta_1 \frac{\partial T}{\partial z} - \beta_2 \frac{\partial C}{\partial z}.
\]

(13)

The governing equations can be put in a more convenient form by using the following non-dimensional variables

\[
(x', z') = c_1 \eta(x, z), \quad (u_1', u_3') = c_1 \eta(u_1, u_3),
\]

\[
t' = c_1^2 \eta t, \quad P' = \frac{P}{\beta_2}, \quad C' = \frac{\beta_2 C}{(\lambda + 2\mu)},
\]

\[
\theta' = \frac{\beta_1 (T - T_0)}{(\lambda + 2\mu)}, \quad \tau_0' = c_1^2 \eta \tau_0, \quad \tau' = c_1^2 \eta \tau,
\]

\[
\sigma_{ij}' = \frac{\sigma_{ij}}{(\lambda + 2\mu)}.
\]

(14)

Where,

\[
\eta = \frac{\rho C_E}{K}, \quad c_1^2 = \frac{(\lambda + 2\mu)}{\rho}.
\]

By using Eqs. (14) in Eqs. (2), (4), (12) and (13) take the form where we have dropped the dash for convenience

\[
\left[ \ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3 \right] = \frac{1}{\beta^2} \nabla^2 \ddot{u}_1 + \left( 1 - \frac{1}{\beta^2} \right) \frac{\partial e}{\partial x} - \frac{\partial T}{\partial x} - \frac{\partial C}{\partial x},
\]

(15)

\[
\left[ \ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1 \right] = \frac{1}{\beta^2} \nabla^2 \ddot{u}_3 + \left( 1 - \frac{1}{\beta^2} \right) \frac{\partial e}{\partial z} - \frac{\partial T}{\partial z} - \frac{\partial C}{\partial z},
\]

(16)

\[
[V^2 - (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})] \theta = \epsilon_1 (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) e_{kk} + \alpha_1 (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) C,
\]

(17)
\( \nabla^2 \varepsilon + \alpha_1 \nabla^2 \theta + \alpha_2 \left( \frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C - \alpha_3 \nabla^2 C = 0. \)  

To solve the problem, we introduce the potential \( \phi(x, z, t) \) and \( \psi(x, z, t) \) through the relations

\[
\begin{align*}
u_1 &= \phi_{,x} + \psi_{,z}, & \quad \nu_3 &= \phi_{,z} - \psi_{,x}.
\end{align*}
\]

Where,

\[
\nabla^2 \psi = \nu_{1,z} - \nu_{3,x}, \quad \epsilon_{kk} = \nabla^2 \varphi.
\]

From Eq. (19) in Eqs. (15)-(18),

\[
(\nabla^2 + \Omega^2 - \frac{\partial^2}{\partial t^2}) \varphi = -2 \Omega \frac{\partial \psi}{\partial t} + \theta + C,
\]

\[
[\nabla^2 - \beta^2 \left( \frac{\partial^2}{\partial t^2} - \Omega^2 \right) ] \psi = 2 \Omega \beta^2 \frac{\partial \phi}{\partial t}.
\]

\[
[\nabla^2 - \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) ] \theta =
\]

\[
(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} ) [\epsilon \nabla^2 \varphi + \alpha_1 C]
\]

\[
\nabla^4 \varphi + \alpha_1 \nabla^2 \theta + \alpha_2 \left( \frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C - \alpha_3 \nabla^2 C = 0
\]

Where \( \beta^2 = (\lambda + 2 \mu) \mu \), \( \epsilon_1 = -\frac{\beta^2 T_0}{\eta K(\lambda + 2 \mu)} \),

\[
\alpha_1 = \frac{a T_0 \beta_1}{\eta K \beta_2}, \quad \alpha_4 = \frac{a (\lambda + 2 \mu)}{\beta_1 \beta_2}, \quad \alpha_2 = \frac{\lambda + 2 \mu}{\beta_2^2},
\]

\[
\alpha_3 = \frac{(\lambda + 2 \mu) b}{\beta_2^2}.
\]

Also Eqs. (6)-(11) take the form

\[
\sigma_{xx} = \frac{2}{\beta^2} \frac{\partial u_1}{\partial x} + (1 - \frac{2}{\beta^2}) \nabla^2 \varphi - \theta - C,
\]

\[
\sigma_{xz} = \frac{1}{\beta^2} \left( \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} \right),
\]

\[
\sigma_{yy} = (1 - \frac{2}{\beta^2}) \nabla^2 \varphi - \theta - C,
\]

\[
P = -\nabla^2 \varphi + \alpha_3 C - \alpha_1 \theta.
\]

III. THE BOUNDARY CONDITIONS OF THE PROBLEM

The boundary conditions are assumed to be

\[
\sigma_{xz}(x, z, t) \big|_{z=0} = f_1(x, t), \quad \sigma_{xz}(x, z, t) \big|_{z=0} = 0,
\]

\[
\theta(x, z, t) \big|_{z=0} = n(x, t), \quad \frac{\partial C}{\partial z} \big|_{z=0} = 0, \quad \frac{\partial \theta}{\partial z} \big|_{z=0} = 0.
\]

Where \( f_1(x, t), n(x, t) \) are known function of \( x \) and \( t \).

IV. THE SOLUATION OF THE PROBLEM

The solution of considered physical variable can be decomposed in terms of normal modes as the following form

\[
\left[ \theta, \varphi, \psi, \sigma_{ij}, C, P \right](x, z, t) = [\theta^*, \varphi^*, \psi^*, \sigma^*_{ij}, C^*, P^*](z) e^{(\iota \omega t + \iota k_x x)},
\]

\[
(D^2 - s_1) \varphi^* = -2 \Omega \omega \varphi^* + \theta^* + C^*,
\]

\[
(D^2 - s_2) \psi^* = s_3 \varphi^*,
\]

\[
(D^2 - k^2)^2 \varphi^* + \alpha_1 (D^2 - k^2) \theta^* + [s_7 - \alpha_3 (D^2 - k^2)] C = 0.
\]

Where, \( D = \frac{d}{dz} \).

Eqs. (32)-(35) form a coupled system of \( \varphi^*, \psi^*, \theta^*, C^* \) and represent coupled thermal.

Eliminating \( \psi^* \) between Eqs. (32) and (33) we obtain
By using Eqs. (40), (41) in Eq. (19) we can obtain

\[ \sigma_{zz}^* = \frac{1}{\beta^2} (\partial u_3^* \partial z) + i k u_3^* \] (47)

\[ \sigma_{xx}^* = 1 \frac{1}{\beta^2} (\partial u_1^* \partial z) + i k u_3^* \] (48)

\[ \sigma_{yy}^* = (1 - \frac{2}{\beta^2}) (i k u_1^* + \partial u_3^*) - \theta^* - C^* \] (49)

\[ P^* = (i k u_1^* + \partial u_3^*) - \alpha_1 \theta^* + \alpha_3 C^* \] (50)

From Eqs. (42)-(45) in Eqs. (46)-(50) we obtain the components of stress tensor

\[ \sigma_{xx}^* = \sum_{i=1}^{5} [(1 - \frac{2}{\beta^2}) k_i^2 - k_i^2 - \frac{2 i k k_i}{\beta^2} R_i - H_i - V_i] G_i e^{-k_i z} \] (51)

\[ \sigma_{zz}^* = \sum_{i=1}^{5} [k_i^2 - (1 - \frac{2}{\beta^2}) k_i^2 + \frac{2 i k k_i}{\beta^2} R_i - H_i - V_i] G_i e^{-k_i z} \] (52)

\[ \sigma_{xx}^* = \sum_{i=1}^{5} \frac{1}{\beta^2} [k_i^2 - k_i^2 - \frac{2 i k k_i}{\beta^2} R_i - 2 i k k_i] G_i e^{-k_i z} \] (53)

\[ \sigma_{yy}^* = \sum_{i=1}^{5} [(1 - \frac{2}{\beta^2}) (k_i^2 - k_i^2) - H_i - V_i] G_i e^{-k_i z} \] (54)

\[ P^* = \sum_{i=1}^{5} [(k_i^2 - k_i^2) - \alpha_1 H_i + \alpha_3 V_i] G_i e^{-k_i z} \] (55)

In order to determine the parameters \( G_i (i = 1, 2, 3, 4, 5) \) we consider the following boundary condition at \( z = 0 \).

By substituting from Eqs. (42), (43), (52) and (53) in Eq. (30) we have

\[ \sum_{i=1}^{5} A_i G_i = f_1^* \] (56)

\[ \sum_{i=1}^{5} B_i G_i = 0 \] (57)

\[ \sum_{i=1}^{5} H_i G_i = n^* \] (58)

\[ \sum_{i=1}^{5} k_i H_i G_i = 0 \] (59)

\[ \sum_{i=1}^{5} k_i V_i G_i = 0 \] (60)

Where, \( A_i, B_i \) are defined in the appendix.
By solving Eqs. (56)-(60) we get the parameters \(G_i (i = 1,2,3,4,5)\).

V. SPECIAL CASE

In the case without rotation we can obtain the following solutions

\[
\varphi^* (z) = \sum_{i=1}^{3} g_i e^{-m_i z},
\]

(61)

\[
\theta^* (z) = \sum_{i=1}^{3} N_i g_i e^{-m_i z},
\]

(62)

\[
C^* (z) = \sum_{i=1}^{3} M_i g_i e^{-m_i z}.
\]

(63)

\[
\psi^* (z) = Be^{-mz}.
\]

(64)

Where \(g_i\) are some parameters, \(N_i, M_i\) are defined in the appendix.

Using Eqs. (61), (64) in Eq. (19) we can obtain

\[
u_1^* = \sum_{i=1}^{3} i k g_i e^{-m_i z} - m B e^{-mz},
\]

(65)

\[
u_3^* = -\sum_{i=1}^{3} m_i g_i e^{-m_i z} - i k Be^{-mz}.
\]

(66)

By using Eqs. (62)-(66) in Eqs. (46)-(50) we obtain the components of stress tensor and the chemical potential

\[
\sigma_{xx}^* = \sum_{i=1}^{3} [(1 - \frac{2}{\beta^2}) m_i^2 - k^2 - N_i - M_i] g_i e^{-m_i z} - \frac{2ikm}{\beta^2} Be^{-mz},
\]

(67)

\[
\sigma_{zz}^* = \sum_{i=1}^{3} [m_i^2 - (1 - \frac{2}{\beta^2}) k^2 - N_i - M_i] g_i e^{-m_i z} + \frac{2ikm}{\beta^2} Be^{-mz},
\]

(68)

\[
\sigma_{yy}^* = \sum_{i=1}^{3} (1 - \frac{2}{\beta^2}) [(m_i^2 - k^2) - N_i - M_i] g_i e^{-m_i z},
\]

(69)

\[
\sigma_{xz}^* = \frac{1}{\beta^2} \left[ -\sum_{i=1}^{3} 2ikm_i g_i e^{-m_i z} + (k^2 + m^2) Be^{-mz} \right],
\]

(70)

\[
P^* = \sum_{i=1}^{3} [(k^2 - m_i^2) - \alpha_1 N_i + \alpha_3 M_i] g_i e^{-m_i z}.
\]

(71)

In order to determine the parameters \(g_i (i = 1,2,3)\) and \(B\), we consider the following boundary condition at \(z = 0\).

Substituting from Eqs. (62), (63), (68) and (70) in Eq. (30) we have

\[
\sum_{i=1}^{3} E_i g_i + E_4 B = f_1^*,
\]

(72)

\[
\sum_{i=1}^{3} F_i g_i + F_4 B = 0,
\]

(73)

\[
\sum_{i=1}^{3} N_i g_i = n^*,
\]

(74)

\[
\sum_{i=1}^{3} m_i M_i g_i = 0.
\]

(75)

Where \(E_i, F_i\) are defined in the appendix.

By solving Eqs. (72)-(75) we get the parameters \(g_i (i = 1,2,3)\) and \(B\).

VI. NUMERICAL RESULTS

The copper material was chosen for purposes of numerical evaluations. The material constants of the problem are thus given by Thomas (1980)

\[
T_0 = 293K, \quad \sigma = 8954 \text{Kg/m}^3, \quad \tau_0 = 0.1s,
\]

\[
\tau = 0.2s, \quad C_E = 383.1 \text{J/(Kg K)},
\]

\[
\alpha_t = 1.78(10)^{-5} \text{K}^{-1}, \quad K = 386 \text{ W/(mK)},
\]

\[
\lambda = 7.76(10)^{10} \text{Kg/((ms}^2), \quad \mu = 3.86(10)^{10} \text{Kg/((ms}^2),
\]

\[
\rho = 8.950(10)^3 \text{Kg/m}^3, \quad \alpha_c = 1.98(10)^{-4} \text{m}^3/\text{Kg},
\]

\[
d = 0.85(10)^{-8} \text{Kg s/m}^3, \quad a = 1.2(10)^4 \text{m}^2/(s^2 K),
\]

\[
b = 0.9(10)^6 \text{m}^5/(\text{Kg s}^2).
\]

Using these values, it was found that

\[
\eta = 8886.73, \quad \epsilon_4 = 0.0168, \quad \beta^2 = 4, \quad \alpha_4 = 5.43,
\]

\[
\alpha_2 = 0.533 \quad \text{and} \quad \alpha_3 = 36.24.
\]

Fig. 1 shows that the relaxation time has a decreasing effect on temperature in the case of \(\Omega = 0\) (without rotation) while in
case of $\Omega = 0.2$ (with rotation) all the curves are coincides. In the two cases we observed that the curves are converges to zero.

Fig. 2 depicts that in the two cases ($\Omega = 0$, $\Omega = 0.2$) the relaxation time has a decreasing effect in the range $0 \leq z \leq 0.7$ while has an increasing effect in the range $z \geq 0.7$. Also we can observed that the magnitude of the displacement with $\Omega = 0.2$ is greater than that for $\Omega = 0$ in the range $0 \leq z \leq 0.7$ but for $z \geq 0.7$ the magnitude of the displacement for $\Omega = 0.2$ is less than that for $\Omega = 0$.

Fig. 3 shows that the relaxation time has a decreasing effect on the stress component $\sigma_{zz}$ in the presence and absence of rotation .we observed also that the magnitude of the stress component $\sigma_{zz}$ in case of $\Omega = 0.2$ is greater than that for $\Omega = 0$ in the range $z \geq 1$.

Fig. 4 shows that as the relaxation time increases the magnitude of the concentration increases in the presence and absence of rotation. Also we can observe that the rotation has a decreasing effect. All the curves in the two cases $\Omega = 0$ and $\Omega = 0.2$ are converges to zero as the distance $z$ increases.

VII. CONCLUSION

Effect of diffusion plays important role in the deformation of the body. The curves exhibit the properties of thermo-diffusivity of the medium, one relaxation time and satisfy the requisite conditions of the problem. It is observed that the rotation has a significant effect on temperature distribution, normal displacement, normal stress and concentration distribution. The results of this problem are very useful in the two-dimensional problem of dynamic response due to various sources of the thermoelastic diffusion. Many studies of the various geophysical and industrial applications of phenomenon of thermoelastic diffusion are used to improve the conditions of elastic solids.

APPENDIX

\[ s_1 = k^2 + \omega^2 - \Omega^2, \quad s_2 = k^2 + \beta^2(\omega^2 - \Omega^2), \]
\[ s_3 = 2\Omega \omega \beta^2, \quad s_4 = k^2 + (\omega + \tau_0 \omega^2), \]
\[ s_5 = \varepsilon_1(\omega + \tau_0 \omega^2), \quad s_6 = a_1(\omega + \tau_0 \omega^2), \]
\[ s_7 = \alpha_2(\omega + \tau_0 \omega^2). \]
\[ R_i = \frac{s_3}{(k_i^2 - s_2)}, \]
\[ H_i = \frac{\left[ s_6(k_i^2 - s_1)(k_i^2 - s_2) + 2\Omega \omega s_3s_6 \right]}{(k_i^2 - s_2)(k_i^2 - s_4 + s_6)} + \frac{s_5(k_i^2 - s_2)(k_i^2 - k^2)}{(k_i^2 - s_2)(k_i^2 - s_4 + s_6)}, \]
\[ V_i = \frac{\left[ (k_i^2 - s_1)(k_i^2 - s_2)(k_i^2 - s_4) + 2\Omega \omega s_5(k_i^2 - s_4 - s_5(k_i^2 - s_2)(k_i^2 - k^2) \right]}{(k_i^2 - s_2)(k_i^2 - s_4 + s_6)}, \]
\[ A_i = \sum_{i=1}^{5} \left[ k_i^2 - \left(1 - \frac{2}{\beta^2}\right)k^2 + \frac{2i\kappa k_i}{\beta^2}R_i - H_i - V_i \right], \]
\[ B_i = \frac{1}{\beta^2} \sum_{i=1}^{5} \left[ (k_i^2 + k^2)R_i - 2i\kappa k_i \right]. \]

In case without rotation:
\[ N_i = \frac{\left[ s_4(k_i^2 - s_1) + s_2(k_i^2 - k^2) \right]}{(k_i^2 - s_3 + s_4)}, \]
\[ M_i = \frac{[k_i^2 - s_1](k_i^2 - s_3 - s_3(k_i^2 - k^2)]}{(k_i^2 - s_3 + s_4)}, \]
\[ F_4 = \frac{i\beta^2(k^2 + m^2)}{2k}, \]
\[ E_i = \sum_{i=1}^{5} \left[ m_i^2 - \left(1 - \frac{2}{\beta^2}\right)k^2 - N_i - M_i \right], \ i = 1,2,3, \]
\[ E_4 = \frac{2i\kappa m}{\beta^2}, \quad F_i = \sum_{i=1}^{5} m_i, \ i = 1,2,3, \]

REFERENCES


