Dynamic Problem of Generalized Thermoelasticity for a Semi-infinite Cylinder with Heat Sources.

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Abstract — In this paper, we discuss the temperature distribution and thermal stresses in a semi-infinite cylinder whose lower and upper surfaces are free of traction and subjected to a given axisymmetric temperature distribution with the help of Lord-Shulman theory and Classical coupled theory of thermoelasticity using integral transform technique. First an exact solution has been obtained in the transform domain. Then the Hankel transforms are inverted analytically and for the inversion of Laplace transforms we apply numerical methods. We have discussed the thermal stresses for a copper material plate and compared the results for both the theories.

Index Terms — Classical coupled theory, Lord-Shulman theory, Laplace transform.

I. INTRODUCTION

The theory of dynamic thermoelasticity has aroused much interest in recent times. It has found applications in many engineering fields such as nuclear reactor design, high energy particle accelerators, geothermal engineering, etc. The heat conduction of classical coupled theory of thermoelasticity is parabolic in nature and hence predicts infinite speed of heat propagation of heat waves. Clearly, this contradicts the physical observations. In the last three decades, focus is on the theories which admit a finite speed for thermal signals. These theories involve a hyperbolic heat equation. The generalization of the classical coupled thermoelasticity theory is due to Biot [1]. The equations of generalized thermoelasticity with one relaxation time were obtained by Lord and Shulman [2] for the isotropic case and by Dhaliwal and Sherief [3] for the anisotropic case. Since the governing equations are complex and the mathematical difficulties associated with the solution, several simplifying assumptions were made. For example, many authors use quasi static equations neglecting the inertia term in the equations of motion [4,5]. These assumptions are in good agreement for many practical applications. However, for a rigorous treatment of problems containing very short time effects or steep heat gradients, the complete system of generalized equations must be utilized. The generalized theory involving two relaxation times was developed by Green and Lindsay (G-L) [6]. Due to the experimental validation available in favour of the finite speed of propagation of heat, generalized thermoelasticity theory is receiving serious attention. Chandrasekariah [7] has studied the development of the second sound effect. Youssef [8, 9, 10, 11] has discussed many important problems in generalized thermoelasticity with various heat sources. Pawar et.al. [12] studied the problem on thermal stresses in an infinite body due to the application of a continuous point heat source. Mallik and Kanoria [13] studied the two dimensional problem in generalized thermoelasticity of thermoelastic interaction for a transversely isotropic thick plate having heat source. These problems are solved using eigen value approach. The state space approach was developed by Sherief. et.al. [14] for two dimensional problems. A two dimensional problem for a half space and for a thick circular plate with heat sources have been solved by El-Maghraby [15, 16]. McDonald [17] studied the importance of thermal diffusion to the thermoelastic wave generation. Bagri and Eslami [18] have got the unified generalized thermoelastic solution for cylinders and spheres. Aouadi [19] studied the discontinuities in an axisymmetric generalized thermoelastic problem.

In the present problem we have modified the work of Aouadi [19] with heat source. The effects of the induced temperature and heat source on the temperature distribution and stress fields in a homogeneous isotropic thermoelastic thick cylinder of height $2h$ and infinite extent have been studied. The analytic solutions are found in Laplace transform domain. Then numerical methods are used to invert the Laplace transforms and to evaluate the integrals involved, so as to obtain the solution in the physical domain. The derived expressions are computed numerically for copper material and the results are presented graphically.

II. FORMULATION OF THE PROBLEM:

Consider an axisymmetric homogeneous isotropic thick plate of height $2h$ defined as $0 \leq r \leq \infty, -h \leq z \leq h$. We take the axis of symmetry as the $z$ axis and the origin of the system of co-ordinates at the middle plane between the upper and lower faces of the plate. The problem is studied using the cylindrical polar co-ordinates $(r, \phi, z)$. Due to the rotational symmetry...
about the $z$ axis of the problem all quantities are independent of the co-ordinate $\phi$.

The displacement vector, thus, has the form

$$\ddot{u} = (u, 0, w)$$

The equations of motion can be written as [20]

$$\mu \nabla^2 u - \frac{\mu}{r^2} u + (\lambda + \mu) \frac{\partial e}{\partial r} - \gamma \frac{\partial T}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2}$$

(1)

$$\mu \nabla^2 w + (\lambda + \mu) \frac{\partial e}{\partial z} - \gamma \frac{\partial T}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}$$

(2)

The generalized equation of heat conduction has the form [20]

$$k \nabla^2 T = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \rho c_E T + \gamma T_0 e \right)$$

$$- \rho \left( 1 + \tau_0 \frac{\partial}{\partial t} \right) Q$$

(3)

where $T$ is the absolute temperature and $e$ is the cubical dilatation given by the relation [20]

$$e = \frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z}$$

(4)

The following constitutive relations supplement the above relations.

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

(5)

$$\sigma_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda e - \gamma (T - T_0)$$

(6)

$$\sigma_{zz} = 2\mu \frac{\partial w}{\partial z} + \lambda e - \gamma (T - T_0)$$

(7)

$$\sigma_{rz} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)$$

(8)

We shall use the following non-dimensional variables

$$r' = c_1 \eta r, \quad z' = c_1 \eta z, \quad u' = c_1 \eta u, \quad w' = c_1 \eta w, \quad t' = c_1^2 \eta t, \quad \tau_0' = c_1^2 \eta \tau_0, \quad \sigma' = \frac{\sigma}{\mu}, \quad \theta = \frac{\gamma (T - T_0)}{(\lambda + 2\mu)}.$$

Using the above non-dimensional variables, the governing equations take the form (dropping the primes for convenience)

$$\nabla^2 u - \frac{u}{r^2} + (\beta^2 - 1) e - \beta^2 \frac{\partial \theta}{\partial r} = \beta^2 \frac{\partial^2 u}{\partial t^2}$$

(9)

$$\nabla^2 w + (\beta^2 - 1) \frac{\partial e}{\partial z} - \beta^2 \frac{\partial \theta}{\partial z} = \beta^2 \frac{\partial^2 w}{\partial t^2}$$

(10)

$$\nabla^2 \theta = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \theta + e \right)$$

$$- \left( 1 + \tau_0 \frac{\partial}{\partial t} \right) \beta^2 Q$$

(11)

while the constitutive relations (6)-(8), becomes

$$\sigma_{rr} = 2 \frac{\partial u}{\partial r} + (\beta^2 - 2) e - \beta^2 \theta$$

(12)

$$\sigma_{zz} = 2 \frac{\partial w}{\partial z} + (\beta^2 - 2) e - \beta^2 \theta$$

(13)

$$\sigma_{rz} = \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)$$

(14)

We note that the equation (4) retains the form

Also $\beta^2 = \frac{(\lambda + 2\mu)}{\mu}.$

Combining equations (9) and (11), we obtain upon using equation (5),

$$\nabla^2 e - \nabla^2 \theta = \frac{\partial^2 e}{\partial t^2}$$

(15)

We assume that the initial state is quiescent, that is, all the initial conditions of the problem are homogeneous.

The boundary conditions are taken as
\[ \theta = f(r,t) \quad , \quad z = \pm h \]  
\[ \sigma_{zz} = \sigma_{rr} = 0 \quad , \quad z = \pm h \]

Where \( f(r,t) \), is a known function of \( r \) and \( t \).

### III. Solution in the Transformed Domain:

Applying the Laplace transform defined by the relation,

\[ \tilde{f}(r,z,s) = L \left[ f(r,z,t) \right] = \int_0^\infty e^{-st} f(r,z,t) dt \]

(17)

to all the equations (9)-(15), we obtain,

\[ \nabla^2 \tilde{\sigma} - \frac{\tilde{u}}{r} + (\beta^2 - 1)\tilde{e} - \beta^2 \frac{\partial \tilde{\sigma}}{\partial r} = \beta^2 s^2 \tilde{u} \]

(18)

\[ \nabla^2 \tilde{w} + (\beta^2 - 1)\frac{\partial \tilde{e}}{\partial z} - \beta^2 \frac{\partial \tilde{\sigma}}{\partial z} = \beta^2 s^2 \tilde{w} \]

(19)

\[ \left( \nabla^2 - s^2 \right) \tilde{\sigma} = (1 + \tau_0 s) (\epsilon s \tilde{e} - \tilde{Q}) \]

(20)

\[ \tilde{\sigma} = \nabla^2 \tilde{\sigma} \]

(21)

The boundary conditions (17), in the transformed domain, take the form

\[ \tilde{\sigma} = \tilde{f}(r,s) \quad , \quad z = \pm h \]

(25)

\[ \sigma_{zz} = \sigma_{rr} = 0 \quad , \quad z = \pm h \]

Eliminating \( \tilde{e} \) between the equations (20) and (21), we get,

\[ \left\{ \nabla^4 - \left( s^2 + s(1 + \tau_0 s)(1 + \epsilon) \right) \nabla^2 + s^3 (1 + \tau_0 s) \right\} \tilde{\sigma} = -(1 + \tau_0 s)(\nabla^2 - s^2)\tilde{Q} \]

(26)

After factorization the above equation becomes,

\[ \left( \nabla^2 - k_1^2 \right) \left( \nabla^2 - k_2^2 \right) \tilde{\sigma} \]

(27)

where \( k_1^2 \) and \( k_2^2 \) are the roots with positive real parts of the characteristic equation

\[ k^4 - \left( s^2 + s(1 + \tau_0 s)(1 + \epsilon) \right) k^2 + s^3 (1 + \tau_0 s) = 0 \]

(28)

The solution of Eq. (27) can be written in the form, 

\[ \tilde{\sigma} = \tilde{\sigma}_1 + \tilde{\sigma}_2 + \tilde{\sigma}_p \]

(29)

where \( \tilde{\sigma}_1 \) is a solution of the homogenous equation,

\[ \left( \nabla^2 - k_i^2 \right) \tilde{\sigma}_i = 0 \quad , i = 1, 2. \]

(30)

and \( \tilde{\sigma}_p \) is a particular solution of equation (27)

In order to solve the problem, we shall use the Hankel transform of order zero with respect to \( r \). This transform of a function \( \tilde{f}(r,z,s) \) is defined by the relation,

\[ \tilde{f}^*(\alpha, z, s) = H \left[ \tilde{f}(r, z, s) \right] = \int_0^\infty \tilde{f}(r, z, s) r J_0(\alpha r) dr \]

(31)

The inverse Hankel transform is given by the relation

\[ \tilde{f}(r, z, s) = H^{-1} \left[ \tilde{f}^*(\alpha, z, s) \right] \]

(32)

Applying the Hankel transform to equation (30), we get

\[ \left( D^2 - \left( k_1^2 + \alpha^2 \right) \right) \tilde{\sigma}_i = 0 \quad , i = 1, 2. \]

where \( D = \partial / \partial z \)

The solution of the above problem can be written in the form,

\[ \tilde{\sigma}_i = A_i(\alpha, s) \left( k_1^2 - s^2 \right) \cosh(q_i z) \]

(33)

where \( q_i = \sqrt{\alpha^2 + k_i^2} \)

Applying the Hankel transform to both sides of equation (27), we get

\[ \left( D^2 - q_i^2 \right) \left( D^2 - q_2^2 \right) \tilde{\sigma}_p^* = -(1 + \tau_0 s)(D^2 - q^2)\tilde{Q}^* \]

(34)

where \( q = \sqrt{\alpha^2 + s^2} \)

We take the heat source \( Q(r, z, t) \) in the following form

\[ Q(r, z, t) = \frac{\delta(t) \delta(r) \cosh z}{2\pi r} \]

(35)

This is a cylindrical shell heat source releasing heat instantaneously at \( t = 0 \) and situated at the centre \( r = 0 \) varying in the axial direction.
On taking Laplace transform and Hankel transform, we get,
\[ \mathcal{Q}^* = \cosh z \tag{36} \]

The solution of the equation (35) has the form,
\[ \mathcal{Q}_\theta^* = \frac{1 - r_0 s}{1 - q_i^2} \left( 1 - q_i^2 \right) \cosh z \tag{37} \]

Then the complete solution in the transformed domain can be written as
\[ \mathcal{Q}_\theta^* (\alpha, z, s) = \sum_{i=1}^{2} A_i (\alpha, s) \left( k_i^2 - s^2 \right) \cosh q_i z \]
\[ - \frac{(1 + r_0 s)(1 - q_i^2)}{(1 - q_i^2)(1 - q_2^2)} \cosh z \tag{38} \]

On taking the inverse Hankel transform of both sides, we get,
\[ \mathcal{Q} (r, z, s) = \int_0^\infty \left[ \sum_{i=1}^{2} A_i (\alpha, s) \left( k_i^2 - s^2 \right) \cosh q_i z \right] \alpha J_0 (\alpha r) d\alpha \tag{39} \]

Similarly eliminating \( \theta \) between equations (20) and (21), we get,
\[ \left( \nabla^2 - k_i^2 \right) \left( \nabla^2 - k_i^2 \right) \mathcal{E}^* = -(1 + r_0 s) \nabla^2 \mathcal{Q} \tag{40} \]

On taking Hankel transform of equation (40), we get,
\[ \left( D^2 - q_i^2 \right) \left( D^2 - q_i^2 \right) \mathcal{E}^* = -(1 + r_0 s) \left( D^2 - \alpha^2 \right) \mathcal{Q}^* \tag{41} \]

Complete solution of equation (41) is of the form,
\[ \mathcal{E}^* (\alpha, z, s) = \sum_{i=1}^{2} A_i (\alpha, s) k_i^2 \cosh q_i z \]
\[ - \frac{(1 + r_0 s)(1 - \alpha^2)}{(1 - q_i^2)(1 - q_2^2)} \cosh z \tag{42} \]

Taking the inverse Hankel transform, we get,
\[ \mathcal{E} (\alpha, z, s) = \int_0^\infty \left[ \sum_{i=1}^{2} A_i (\alpha, s) k_i^2 \cosh q_i z \right] \alpha J_0 (\alpha r) d\alpha \tag{43} \]

Taking Hankel transform of equation (20) and using equations (40) and (43), the complete solution can be written as,
\[ \overline{w}^* (\alpha, z, s) = B (\alpha, s) \sinh q_3 z \]
\[ + \sum_{i=1}^{2} A_i (\alpha, s) q_i \sinh q_i z \tag{44} \]
\[ - \frac{(1 + r_0 s)}{(1 - q_i^2)(1 - q_2^2)} \sinh z \]

where \( q_3 = \sqrt{\alpha^2 + \beta^2 s^2} \)

On inverting the Hankel transform, we get,
\[ \overline{w} (\alpha, z, s) = \int_0^\infty \left[ \sum_{i=1}^{2} A_i (\alpha, s) q_i \sinh q_i z \right] \alpha J_0 (\alpha r) d\alpha \tag{45} \]

Taking the Hankel and Laplace transform of both sides of equation (4) and using equations (43) and (45), we get,
\[ H \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \overline{u}) \right] = - \alpha^2 \sum_{i=1}^{2} A_i (\alpha, s) \cosh q_i z \]
\[ - \frac{(1 + r_0 s)}{(1 - q_i^2)(1 - q_2^2)} \cosh z \tag{46} \]

On applying inverse Hankel Transform on both sides of equation (46), we get,
\[ \overline{u} = \int_0^\infty \left\{ \sum_{i=1}^{2} A_i (\alpha, s) \cosh q_i z \right\} \alpha J_0 (\alpha r) d\alpha \tag{47} \]

The stress tensor components are in the form
\[ \overline{\sigma} = \int_0^\infty \left\{ \sum_{i=1}^{2} A_i (\alpha, s) \cosh q_i z \right\} \alpha J_0 (\alpha r) d\alpha \tag{48} \]
\[ \bar{\sigma}_{zz} = \left\{ \begin{array}{l} \frac{-\left(\alpha^2 + q_r^2\right)B(\alpha, s) \cosh q_r z}{2} \\ \sum_{i=1}^{2} A_i(\alpha, s) q_i \sinh q_i z \\ -2 \alpha^2 \left( \frac{1 + \tau_0 s}{1 - q_i^2} \right) \sinh(z) \end{array} \right. \] 

After applying the Hankel transform, the boundary conditions become,

\[ \bar{\sigma}_{zz}^*(\alpha, z, s) = \tilde{f}^*(\alpha, s), \quad z = \pm h \]

\[ \bar{\sigma}_{zz} = 0, \quad z = \pm h \]

On applying the boundary conditions (50) and (51) to determine the unknown parameters, we get,

\[ \sum_{i=1}^{2} A_i(\alpha, s) \left( \alpha^2 + q_r^2 \right) \cosh q_r h \]

\[ - \frac{(1 + \tau_0 s)(1 - q_i^2)}{(1 - q_i^2)(1 - q_r^2)} \cosh(h) = \tilde{f}^*(\alpha, s) \]

\[ \left( \alpha^2 + q_r^2 \right) \sum_{i=1}^{2} A_i(\alpha, s) \cosh q_i h - 2q_3 B(\alpha, s) \cosh q_r h \]

\[ = \frac{(1 + \tau_0 s)(\alpha^2 + q_3^2)}{(1 - q_i^2)(1 - q_r^2)} \cosh(h) \]

\[ 2 \alpha^2 \sum_{i=1}^{2} A_i(\alpha, s) q_i \sinh q_i h + \left( \alpha^2 + q_r^2 \right) B(\alpha, s) \cosh q_r h \]

\[ = \frac{2 \alpha^2 (1 + \tau_0 s)}{(1 - q_i^2)(1 - q_r^2)} \sinh(h) \]

On solving equations (52)-(54) numerically, we get the complete solution of the problem in the transformed domain.

IV. INVERSION OF DOUBLE TRANSFORMS:

Due to the complexity of the solution in the laplace transform domain, the inverse of the Laplace transform is obtained using the Gaver-Stehfast algorithm [21,22,23]. A detailed explanation can be found in Knight and Raiche [24]. The final formula based on the work done by Widder[25] who developed an inversion operator for the Laplace transform is given here. Gaver and Stehfast modified this operator and derived the formula

\[ f(t) = \frac{\ln 2}{t} \sum_{j=1}^{K} D(j, K) F \left( j \frac{\ln 2}{t} \right) \]

with

\[ W(s, K) = (-1)^{K-M} \sum_{n=m}^{M} K = n! (n-1)! (j-n)! (2n-j)! \]

where \( K \) is an even integer, whose value depends on the word length of the computer used. \( M = K/2 \) and \( m \) is the integer part of the \( (j+1)/2 \). The optimal value of \( K \) was chosen as described in Stehfast algorithm, for the fast convergence of results with the desired accuracy. The Romberg numerical integration technique [26] with variable step size was used to evaluate the integrals involved. All the programs were made in mathematical software MATLAB.

V. NUMERICAL RESULTS AND DISCUSSION:

For numerical calculations we take

\[ f(r, t) = \theta_0 H(\alpha - r) H(t) \]

where \( \theta_0 \) is a constant.

On taking Hankel and Laplace transform of the above function, we get,

\[ \tilde{f}^*(\alpha, s) = \frac{a \theta_0 J_1(\alpha a)}{\alpha s} \]

Copper material was chosen for purposes of numerical evaluations. The constants of the problem are shown below

\[ k = 386 J.K^{-1}.m^{-1}.s^{-1} \]

\[ \alpha_i = 1.78 \times 10^{-5} K^{-1} \]

\[ c_E = 383.1 J.K_0^{-1}.s^{-1} \]

\[ \eta = 8886.73 s.m^{-2} \]

\[ \mu = 3.86 \times 10^{10} N.m^{-2} \]

\[ \lambda = 7.76 \times 10^{10} N.m^{-2} \]

\[ \rho = 8954 kg.m^{-3} \]

\[ c_1 = 4.158 \times 10^3 m.s^{-1} \]

\[ \tau_0 = 0.02 s \]

\[ T_0 = 293 K \]

\[ \varepsilon = 0.0168 N.m.J^{-1} \]

\[ \beta^2 = 4 \]

\[ \alpha = 1 \]

\[ \theta_0 = 1 \]

\[ h = 1 \]

The numerical values for temperature \( \theta \), the radial displacement component \( u \), the axial stress component \( \sigma_{zz} \) and the shear stress component \( \sigma_{r\theta} \) have been calculated at the middle of the plane \( z = 0 \) for different time instants \( t = 0.1, 0.25, 1.1 s \) along the radial direction and are displayed graphically for Lord-Shulman theory (L-S theory) and Classical Coupled Thermoelasticity (CT theory) as shown in fig 1,2,3 and 4 respectively. Since the displacement function \( w \) is an odd function of \( z \), its value on the middle plane is always zero and it is not represented graphically here.

Fig 1, shows the non-dimensional temperature distribution along the radial direction at the middle plane \( z = 0 \) at different time instants \( t = 0.1, 0.25, 1.1 s \) . Classical Coupled Theory of thermoelasticity (CT) predicts an infinite speed of wave propagation, whereas the Lord-Shulman (LS) model of generalized thermoelasticity involves the introduction of one relaxation time \( \tau_0 \), due to which the waves assume finite propagation speeds. Hence the variation in
values is clearly seen for the two theories in the plots. But the nature of the curve seems to be the same in both the theories. It is also observed that the non-dimensional temperature $\theta$ drops gradually along the radial direction.

Fig. 2, shows the plot of radial displacement $u$ along the radial direction at the middle of the plane ($z = 0$) at different time instants. It is observed that the radial displacement increases from zero and becomes maximum near $r = 4m$, then it decreases as $r$ increases and becomes again zero near $r = 9m$.

Fig. 3, shows the variation of axial stress $\sigma_{zz}$ along the radial direction in the middle plane ($z = 0$). A difference in profiles of axial stress is seen at small times (i.e. at $t = 0.1, 0.25s$) and large times (i.e. at $t = 1.1s$). The difference in results for LS and CT can also be seen in the plot.

Fig. 4, shows the shear stress $\sigma_{rz}$ distribution along the radial direction of the cylinder in the middle plane ($z = 0$) at different time instants. Shear stress shows sinusoidal nature in the plots with high peaks in the middle of the plane and gradually reducing as the radius increases.

Fig. 5, shows the plots of non-dimensional temperature distribution along $z$ axis at different time instants $t = 0.1, 0.25, 1.1s$ and at $r = 1m$. For both LS and CT theories at small times, the variation in values of non-dimensional temperature is clearly seen for the two theories in the plots. For large times $t = 1.1s$, it can be observed that the plots of temperature versus $z$ coincide for both the theories. Hence LS and CT give similar results for large times.

Fig. 6, shows the plot of shear stress component $\sigma_{rz}$ along $z$ axis at different time instants $t = 0.1, 0.25, 1.1s$ and at $r = 1m$. Graph shows compressive nature of shear stress in the region $z = -1m$ to $z = -0.2m$ and later on the stress becomes tensile. It can be further observed that for large times $t = 1.1s$, both the theories give identically equivalent results.

Fig. 7, shows the plot of axial stress component $\sigma_{zz}$ along $z$ axis at different time instants $t = 0.1, 0.25, 1.1s$ for $r = 1m$. From $z = -1m$ to $z = 0.4m$, tensile nature of the axial stress $\sigma_{zz}$ is predicted and after $z = 0.4m$ stress becomes compressive, i.e. for a small region near the top of the plate, the axial stress is compressive. It can also be observed from the plots of axial stress and Shear stress that the mechanical boundary conditions are satisfied at $z = \pm h$.

Clearly the difference between the LS and CT theory of thermoelasticity is observed in the plots.

VI. CONCLUSION:

In this problem we have used the generalized theory of thermoelasticity (L-S model) to solve the problem for semi-infinite cylinder with heat source and compared the model with Classical coupled theory (C T). We have directly found the solution for the field equations without using the potential functions. This helps in eliminating the well known problems associated with the solutions using potential functions. The numerical inversion methods are very fast and accurate as compared to any other methods. Due to the presence of one relaxation time in the field equations the heat wave assumes finite speed of propagation. From the graphs we can clearly observe that the results obtained using the generalized theory of thermoelasticity (L S model) with one relaxation time are different from the results obtained by using the Classical coupled theory (C T model) of thermoelasticity. We may conclude that the system of equations in this paper may prove to be useful in studying the thermal characteristics of various bodies in important engineering problems using the more realistic Lord-Shulman model of thermoelasticity predicting finite speeds of wave propagation.
Fig. 3. Axial stress component $\sigma_{zz}$ along the radial direction in the middle plane.

Fig. 4. Shear Stress Component $\sigma_{rz}$ along the radial direction in the middle plane.

Fig. 5. Temperature distribution $\theta$ along $z$ axis at $r = 1m$.

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