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2-D Problem of Generalized Magneto-Thermoelastic Diffusion with Temperature-Dependent Elastic Moduli

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***Abstract*— The present investigation is concerned with the effect of magnetic field on the generalized thermoelastic diffusion with temperature-dependent elastic meduli. The modulus of elasticity is taken as a linear function of reference temperature. The present model is describing the generalization Lord-Shulman theory. The expressions for the temperature, displacement components, the thermal stress distributions and the concentration of diffusion are obtained by using the normal mode method. Numerical results are obtained and illustrated graphically for the problem considered.**

Keywords; Magnetic field, Thermoelasticity, Diffusion, temperature-dependent, Normal mode method, Lord-Shulman theory

I. Introduction

Noda [1] reviewed extensive studies on thermal stresses in materials with temperature-dependent properties. Material properties such as modulus of elasticity and thermal conductivity vary with temperature. In the thermal stress analyses of engineering materials and structures, the properties of materials are usually regarded as constants, which is approximately correct when the temperature variation from the initial temperature is not very high. In reactor vessels, turbine engines, space vehicles and refractory industries, the structural components are exposed to high temperature changes. In thermal stress analyses of these components, neglecting the temperature dependence of material properties will usually result in significant errors. Jin and Batra [2] studied the thermal fracture mechanics of ceramic with temperature-dependent properties. Othman [3] used Lord-Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermo-elasticity. Othman et al. [4] investigated the generalized thermoelastic medium with temperature dependent properties for different theories under the effect of gravity field. The state space approach to the generalized thermo-elastic problem with temperature-dependent elastic moduli and internal heat sources is discussed by Othman [5].

Diffusion is defined as a random walk of an ensemble of particles, from regions of high concentration to a region of lower concentration. The study of this phenomenon is due to many applications in geophysics and electronic industry. Diffusion is employed to introduce dopants in controlled amounts into the semi-conductor substance integrated circuit fabrication. Also, in Metal Oxide Semi-conductor (MOS) transistor, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors and the source/drain regions. The conditions of oil extraction are improved by using diffusion. The oil companies used the process of thermo-diffusion to extract oil from oil deposits. The concentration is obtained by using Fick's law [6]. The coupled thermoelastic model is used to develop the theory of thermoelastic diffusion by Nowacki [7-10]. Sherief et al. [11] discussed the theory of generalized thermoelastic diffusion with one relaxation time. This implies a finite speed of propagation of waves. The effect of rotation on a thermoelastic diffusion with temperature-dependent elastic moduli comparison of different theories is studied by Elmaklizi and Othman [12].

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Othman [13] studied the effect of rotation and thermal shock on a perfect conducting elastic half-space in generalized magneto-thermo-elasticity with two relaxation times.

The effect of magnetic field on the elastic wave propagation is very important due to its much application in the field of geophysics, plasma physics. Othman and Lotfy [14] studied the two-dimensional problem of generalized magneto-thermoelasticity with temperature dependent elastic moduli for the different theories. Various authors discussed different types of problems in magneto-thermoelastic medium [15-20].

In the present paper, we will study the effect of magnetic field on thermoelastic diffusion under Lord-Shulman theory in a perfectly conducting medium. The moduli of elasticity are taken as a linear function of reference temperature.

II. Formulation of the Problem

Let us consider an isotropic, homogeneous, thermally and perfectly conducting elastic medium with temperature-dependent of modulus of elasticity. We consider an orthogonal Cartesian coordinate system oxyz having originated on the surface $z=0$ and oz being a line drawn vertically downwards. The medium is subjected to a magnetic field H_0 which is parallel to y-axis. Maxwell 's equations for homogeneous isotropic material (Strictly speaking, when the material is subjected to magnetic fields and thermal field the material will not remain homogeneous and isotropic; this variation is ignored in this investigation) are given by [21]

$$\nabla \wedge \underline{h} = \underline{J} + \varepsilon \frac{\partial \underline{E}}{\partial t}, \tag{1}$$

$$\nabla \wedge \underline{E} = -\mu_e \frac{\partial \underline{h}}{\partial t}, \tag{2}$$

$$\nabla . \underline{h} = 0, \tag{3}$$

and Ohm's law [15] is

$$\underline{J} = \sigma [\underline{E} + \mu_e \frac{\partial \underline{u}}{\partial t} \wedge \underline{h}] - K_0 \nabla T, \tag{4}$$

where $\underline{E}$, $\underline{h}$, $\underline{J}$, μ_e, σ, K_0, $\underline{u}$, T, t and ε are the electric intensity, the magnetic intensity, the current density vector, the magnetic permeability, the electric conductivity the linking electric field with temperature gradient, the displacement vector, the absolute temperature absolute, time and the permittivity.

The elastic medium is perfectly conducting, and then Eq. (4) is reduced to

$$\underline{E} = -\mu_e \frac{\partial \underline{u}}{\partial t} \wedge \underline{h}. \tag{5}$$

The constitutive law of the theory of generalized thermo-elasticity diffusion with one relaxation time is written as [11], [21]

(1) The equation of motion

$$\rho \ddot{u}_i = \sigma_{ij,j} + F_i , \tag{6}$$

where ρ is the density of the medium, σ_{ij} are the components of the stress tensor and F_i is the Lorentz force and taken the form

$$F_i = \mu_e (\underline{J} \wedge \underline{h})_i . \tag{7}$$

(2) The strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) . \tag{8}$$

(3) The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} (\lambda e_{kk} - \nu \theta - \beta C), \tag{9}$$

$$P = -\beta e_{kk} + bC - a\theta, \tag{10}$$

where P is the chemical potential, λ, μ are Lame 's constants, $\nu = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, $\beta = (3\lambda + 2\mu)\alpha_c$, α_c is the coefficient of linear diffusion expansion, a is the measure of thermo-diffusion effect, b is the measure of diffusive effect, C is the concentration of the diffusive material in the elastic medium and $\theta = T - T_0$, T_0 is the temperature of the medium in its natural state assumed to be such that $|\theta / T_0| << 1$.

(4) The energy equation

$$K\theta_{,ii} = \rho C_E (\dot{\theta} + \tau_0 \ddot{\theta}) + \nu T_0 (\dot{e} + \tau_0 \ddot{e}) + aT_0 (\dot{C} + \tau_0 \ddot{C}), \tag{11}$$

where K is the thermal conductivity, τ_0 is the thermal relaxation time and C_E is the specific heat at constant strain.

(5) The diffusion equation

$$d\beta e_{kk,ii} + da\theta_{,ii} + \dot{C} + \tau \ddot{C} - dbC_{,ii} = 0, \tag{12}$$

where d is the diffusion coefficient and τ is the diffusion relaxation time.

We consider all quantities are functions the coordinates x, z and t. The displacement components have the following form

$$u_x = u(x, z, t), \qquad u_y = 0, \qquad u_z = w(x, z, t). \tag{13}$$

Now, we assume that [12]

$$\lambda = \lambda_0 (1 - \alpha^* T_0), \quad \mu = \mu_0 (1 - \alpha^* T_0), \quad \nu = \nu_0 (1 - \alpha^* T_0),$$

$$\beta = \beta_0 (1 - \alpha^* T_0). \tag{14}$$

Where λ_0, μ_0, ν_0, β_0 are constants and α^* is the linear temperature coefficient. In the case of the modulus elasticity is temperature independent $\alpha^* = 0$.

By using Eqs. (5) and (7), we get

$$\underline{E} = \mu_e H_0 (\dot{w}, 0, -\dot{u}), \tag{15}$$

$$\underline{F} = -\mu_e H_0[(\frac{\partial h}{\partial x} + \mu_e \varepsilon \ddot{u}), 0, (\frac{\partial h}{\partial z} + \mu_e \varepsilon \ddot{w})]. \quad (16)$$

By substituting from Eq. (14) in Eqs. (9) and (10), we obtain

$$\alpha_1 \sigma_{xx} = (2\mu_0 + \lambda_0) e_{xx} + \lambda_0 e_{zz} - \nu_0 \theta - \beta_0 C, \quad (17)$$

$$\alpha_1 \sigma_{zz} = (2\mu_0 + \lambda_0) e_{zz} + \lambda_0 e_{xx} - \nu_0 \theta - \beta_0 C, \quad (18)$$

$$\alpha_1 \sigma_{xz} = 2\mu_0 e_{xz}, \quad (19)$$

$$P = -(\beta_0 / \alpha_1) e_{kk} + bC - a\theta, \quad (20)$$

where $\alpha_1 = 1/(1-\alpha^* T_0)$. (21)

By using Eqs. (15)-(19) in Eq. (6), we get

$$\alpha_1 \rho \ddot{u} = (\mu_0 + \lambda_0)\frac{\partial e}{\partial x} + \mu_0 \nabla^2 u - \nu_0 \frac{\partial \theta}{\partial x} - \beta_0 \frac{\partial C}{\partial x} + \alpha_1 \mu_e H_0^2 (\frac{\partial e}{\partial x} - \mu_e \varepsilon \ddot{u}), \quad (22)$$

$$\alpha_1 \rho \ddot{w} = (\mu_0 + \lambda_0)\frac{\partial e}{\partial z} + \mu_0 \nabla^2 u - \nu_0 \frac{\partial \theta}{\partial z} - \beta_0 \frac{\partial C}{\partial z} + \alpha_1 \mu_e H_0^2 (\frac{\partial e}{\partial z} - \mu_e \varepsilon \ddot{w}). \quad (23)$$

By substituting from Eq. (14) in Eqs. (11) and (12), we obtain

$$K \ddot{\theta}_{,ii} = \rho C_E (\dot{\theta} + \tau_0 \ddot{\theta}) + (T_0 \nu_0 / \alpha_1)(\dot{e} + \tau_0 \ddot{e}) + a T_0 (\dot{C} + \tau_0 \ddot{C}), \quad (24)$$

$$(d\beta_0 / \alpha_1) e_{kk,ii} + da\theta_{,ii} + \dot{C} + \tau \ddot{C} - dbC_{,ii} = 0. \quad (25)$$

We will use the following non-dimensional variables

$$(x', z') = \frac{\tilde{\omega}}{c_1}(x, z), \quad t' = \tilde{\omega} t, \quad \tau_0' = \tilde{\omega}\tau_0, \quad \tau' = \tilde{\omega}\tau,$$

$$(u', w') = \frac{\tilde{\omega}}{c_1}(u, w), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\rho c_1^2}, \quad h' = \frac{h}{H_0}, \quad C' = \frac{\beta_0}{\rho c_1^2} C,$$

$$\theta' = \frac{\nu_0 \theta}{\rho c_1^2}, \quad P' = \frac{P}{\beta_0},$$

where $\tilde{\omega} = \frac{\rho C_E c_1^2}{K}$ and $c_1^2 = (\lambda_0 + 2\mu_0)/\rho$.

Using these non-dimensional equations take the form (omitting the primes for convenience)

$$\ddot{u} = \frac{1}{\alpha_1}[\beta_1 \frac{\partial e}{\partial x} + (1-\beta_1)\nabla^2 u - \frac{\partial \theta}{\partial x} - \frac{\partial C}{\partial x}] + A_0 \frac{\partial e}{\partial x} - B_0 \ddot{u}, \quad (26)$$

$$\ddot{w} = \frac{1}{\alpha_1}[\beta_1 \frac{\partial e}{\partial z} + (1-\beta_1)\nabla^2 w - \frac{\partial \theta}{\partial z} - \frac{\partial C}{\partial z}] + A_0 \frac{\partial e}{\partial z} - B_0 \ddot{w}, \quad (27)$$

$$\nabla^2 \theta = (\dot{\theta} + \tau_0 \ddot{\theta}) + (\delta\delta_0 / \alpha_1)(\dot{e} + \tau_0 \ddot{e}) + a_1 \delta_0 (\dot{C} + \tau_0 \ddot{C}), \quad (28)$$

$$\nabla^2 e + \alpha_1 \alpha_2 \nabla^2 \theta + \alpha_1 \alpha_3 (\dot{C} + \tau \ddot{C}) - \alpha_4 \alpha_1 \nabla^2 C = 0, \quad (29)$$

$$\sigma_{xx} = \frac{1}{\alpha_1}[\frac{\partial u}{\partial x} + (2\beta_1 - 1)\frac{\partial w}{\partial z} - \theta - C], \quad (30)$$

$$\sigma_{zz} = \frac{1}{\alpha_1}[\frac{\partial w}{\partial x} + (2\beta_1 - 1)\frac{\partial u}{\partial x} - \theta - C], \quad (31)$$

$$\sigma_{xz} = \frac{1}{\alpha_1}(1-\beta_1)[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}], \quad (32)$$

$$P = -\frac{1}{\alpha_1}(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) + \alpha_4 C - \alpha_2 \theta, \quad (33)$$

Where, $A_0 = \frac{\mu_e H_0^2}{\rho c_1^2}$, $B_0 = \frac{\mu_e^2 \varepsilon H_0^2}{\rho}$, $\delta = \frac{\nu_0}{\rho C_E}$,

$$a_1 = \frac{a c_1^2}{\beta_0 C_E}, \quad \alpha_2 = \frac{a\rho c_1^2}{\beta_0 \nu_0}, \quad \alpha_3 = \frac{c_1^2 K}{d\beta_0^2 C_E}, \quad \alpha_4 = \frac{c_1^2 b\rho}{\beta_0^2},$$

$$\beta_1 = (\mu_0 + \lambda_0)/\rho c_1^2, \quad T_0 = \frac{\rho c_1^2 \delta_0}{\nu_0}.$$

Introducing the potential functions $\phi(x,z,t)$ and $\psi(x,z,t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \quad (34)$$

By substituting Eq. (34) in Eqs. (26)-(29), we get

$$[\alpha \nabla^2 - \alpha_1 \gamma \frac{\partial^2}{\partial t^2}]\phi = \theta + C, \quad (35)$$

$$[\alpha_1 \gamma \frac{\partial^2}{\partial t^2} - (1-\beta_1)\nabla^2]\psi = 0, \quad (36)$$

$$[\nabla^2 - (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})]\theta = \delta_0 (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})[\frac{\delta}{\alpha_1}\nabla^2 \phi + a_1 C], \quad (37)$$

$$\nabla^2 \phi + \alpha_1 [\alpha_2 \nabla^2 \theta + \alpha_3 (\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}) C - \alpha_4 \nabla^2 C] = 0, \quad (38)$$

where $\alpha = 1 + \alpha_1 A_0$ and $\gamma = 1 + B_0$.

On the plane $z = 0$, the boundary conditions are written as follows:

$$\sigma_{xz} = 0, \quad \sigma_{zz} = -F_0 e^{\omega t + ikx}, \quad \frac{\partial \theta}{\partial z} = 0, \quad \frac{\partial C}{\partial z} = 0. \quad (39)$$

III. THE SOLUTION OF THE PROBLEM

The normal mode analysis is used to solve the problem as follows

$$[\theta, \phi, \psi, \sigma_{ij}, C, P](x,z,t) = [\theta^*, \phi^*, \psi^*, \sigma_{ij}^*, C^*, P^*](z)\exp(\omega t + ikx) \quad (40)$$

Substituting from Eq. (40) in Eqs. (35)-(38), we get

$$(\alpha D^2 - s_1)\phi^* = \theta^* + C^*, \quad (41)$$

$$(D^2 - m^2)\psi^* = 0, \quad (42)$$

$$(D^2 - s_2)\theta^* = s_3 (D^2 - k^2)\phi^* + s_4 C^*, \quad (43)$$

$$(D^2 - k^2)^2 \phi^* = -\alpha_1 \alpha_2 (D^2 - k^2)\theta^* + \alpha_1 [\alpha_4 (D^2 - k^2) - s_5] C^*, \quad (44)$$

Where, $s_n (n = 1,2,3,4,5)$ and m^2 are given in the Appendix.

Eliminating θ^* and C^* between Eqs. (41), (43) and (44), we get

$$[D^6 - L_0 D^4 + L_1 D^2 - L_2]\phi^*(z) = 0, \quad (45)$$

where L_0, L_1 and L_2 are defined in the Appendix.
In a similar manner, we obtain

$$[D^6 - L_0 D^4 + L_1 D^2 - L_2](\theta^*, C^*)(z) = 0. \quad (46)$$

Eq. (45) can be factored as

$$(D^2 - K_1^2)(D^2 - K_2^2)(D^2 - K_3^2)\phi^* = 0. \quad (47)$$

The solution Eq. (47) must be bounded as $z \to \infty$. Then, it is written as

$$\phi^* = \sum_{i=1}^{3} G_i e^{-K_i z}. \quad (48)$$

Similarly

$$\theta^* = \sum_{i=1}^{3} H_i G_i e^{-K_i z}, \quad (49)$$

$$C^* = \sum_{i=1}^{3} R_i G_i e^{-K_i z}. \quad (50)$$

Where, G_i are some parameters, K_i^2 are the roots of the characteristic equation of Eq. (47), H_i and R_i are given in the Appendix.
The solution of Eq. (42) is obtained as

$$\psi^* = B e^{-mz}. \quad (51)$$

Substituting from (48) and (51) in (34), we obtain

$$u^* = ik \sum_{i}^{3} G_i e^{-K_i z} + mBe^{-mz}, \quad (52)$$

$$w^* = -\sum_{i}^{3} K_i G_i e^{-K_i z} + ikBe^{-mz}. \quad (53)$$

By substituting from (40) in Eqs. (30)-(33), we get

$$\sigma_{xx}^* = \frac{1}{\alpha_1}[iku^* + (2\beta_1 - 1)Dw^* - \theta^* - C^*], \quad (54)$$

$$\sigma_{zz}^* = \frac{1}{\alpha_1}[Dw^* + ik(2\beta_1 - 1)u^* - \theta^* - C^*], \quad (55)$$

$$\sigma_{xz}^* = \frac{1}{\alpha_1}(1 - \beta_1)[Du^* + ikw^*], \quad (56)$$

$$P^* = -\frac{1}{\alpha_1}(iku^* + Dw^*) + \alpha_4 C^* - \alpha_2 \theta^*. \quad (57)$$

Substituting from Eqs. (49)-(53) in Eqs. (54)-(56), we obtain

$$\sigma_{xx}^* = \frac{1}{\alpha_1}[\sum_{i=1}^{3}\{-k^2 + (2\beta_1 - 1)K_i^2 - H_i - R_i\}G_i e^{-K_i z} + 2ikm(1 - \beta_1)Be^{-mz}], \quad (58)$$

$$\sigma_{zz}^* = \frac{1}{\alpha_1}[\sum_{i=1}^{3}\{K_i^2 - k^2(2\beta_1 - 1) - H_i - R_i\}G_i e^{-K_i z} + 2ikm(\beta_1 - 1)Be^{-mz}], \quad (59)$$

$$\sigma_{xz}^* = \frac{1}{\alpha_1}(1 - \beta_1)[-2ik\sum_{i=1}^{3} K_i G_i e^{-K_i z} - (m^2 + k^2)Be^{-mz}]. \quad (60)$$

Substituting from Eq. (40) in Eqs. (39), we get

$$\sigma_{xz}^* = 0, \quad \sigma_{zz}^* = -F^*, \quad \frac{\partial \theta^*}{\partial z} = 0, \quad \frac{\partial C^*}{\partial z} = 0 \quad \text{at} \quad z = 0. \quad (61)$$

From Eqs. (49), (50), (59) and (60) in Eq. (61), we get

$$i\sum_{i=1}^{3} N_i G_i + N_4 B = 0, \quad (62)$$

$$\sum_{i=1}^{3} M_i G_i + iM_4 B = -F^*, \quad (63)$$

$$\sum_{i=1}^{3} K_i H_i G_i = 0, \quad (64)$$

$$\sum_{i=1}^{3} K_i R_i G_i = 0. \quad (65)$$

Where M_i, M_4, N_i and N_4 are given in the appendix. Solving Eqs. (62)-(65), we obtain the parameters G_i, $(i = 1, 2, 3)$ and B are defined as follows:

$$G_1 = \frac{\Delta_1}{\Delta}, \quad G_2 = \frac{\Delta_2}{\Delta}, \quad G_3 = \frac{\Delta_3}{\Delta}, \quad B = \frac{i\Delta_4}{\Delta}, \quad (66)$$

where $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are given in the Appendix.

IV. Numerical Result and Discussion

In order to explain theoretical results in the preceding sections, we now present some numerical results. We chose the physical data of copper as [11]

$K = 386\,(J/KgK)$, $T_0 = 293K$, $t = 0.1\,s$, $\tau = 0.02\,s$, $\rho = 8954(Kg/m^3)$, $C_E = 383.1$, $\alpha_t = 1.78\text{x}10^{-5}K^{-1}$, $\alpha_c = 1.98\text{x}10^{-4}(m^3/Kg)$, $\lambda_0 = 7.76\text{x}10^{10}\,Kg/(ms^2)$, $\mu_0 = 3.86\text{x}10^{10}\,Kg/(ms^2)$, $d = 0.85\text{x}10^{-8}\,Kgs/m^3$, $a = 1.2\text{x}10^4 m^2/(s^2K)$, $b = 0.9\text{x}10^6 m^5/(Kgs^2)$, $z = 3$, $F^* = 10^5$, $\mu_e = 12.566\text{x}10^{-7}$, $\varepsilon = 8.854\text{x}10^{-12}$, $\tau_0 = 0.025\,s$, $\alpha^* = 0.0012$.

Since ω is complex, then we take $\omega = \omega_0 + i\zeta$, $\omega_0 = 2$ and $\zeta = 1$. The effect of magnetic field on the temperature

θ, the components of displacements u, w the components of stresses $\sigma_{xx}, \sigma_{zz}, \sigma_{xz}$, concentration c and chemical potential P are given in Figs. 1-8, when $\alpha = 1$, ($i.e.$ $H_0 = 0$) and $\alpha = 1.1$ ($i.e.$ $H_0 = 10^8$). In all figures the modulus of elasticity id dependent on temperature. Also the curves which are marked with the symbols 0 and x are represented $\alpha = 1$ and $\alpha = 1.1$ respectively. In all figures, it's shown that the magnetic field has an increasing effect and a decreasing effect in three ranges.

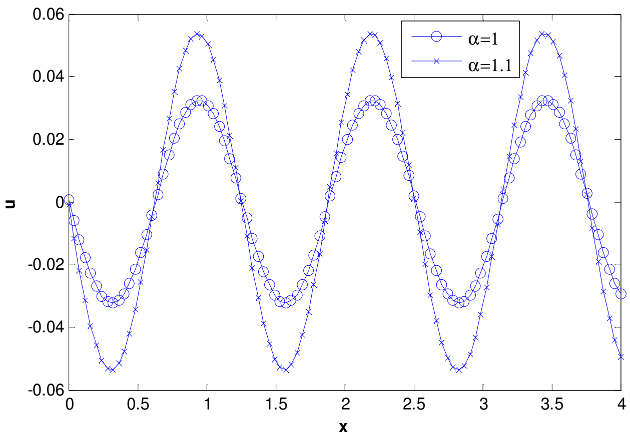

Fig. 1 Variation of the displacement component u versus

Fig. 1 shows the variation of the displacement component u versus x. The magnetic field has an increasing effect in the ranges $0.6 \le x \le 1.2$, $1.9 \le x \le 2.5$ and $3.2 \le x \le 3.8$. It has a decreasing effect in the ranges $0 \le x \le 0.6$, $1.2 \le x \le 1.9$ and $2.5 \le x \le 3.2$.

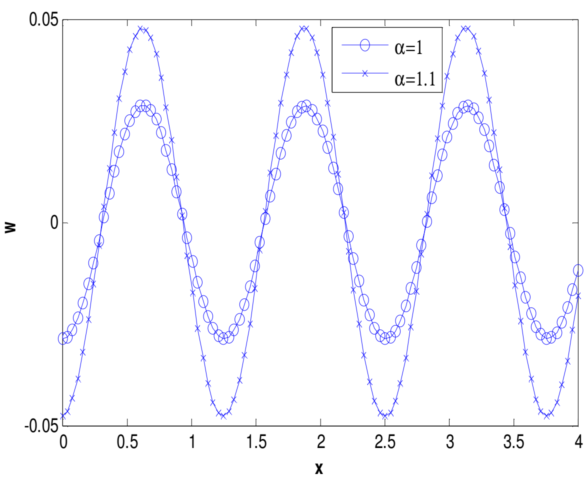

Fig. 2 Variation of the displacement component w versus x

Fig. 2 represents the variation of the displacement component w. The values of w for $\alpha = 1.1$ are large compared to those for $\alpha = 1$ in the ranges $0.3 \le x \le 0.96$, $1.6 \le x \le 2.2$ and $2.8 \le x \le 3.5$, but the amplitude of w for $\alpha = 1.1$ is smaller than that for $\alpha = 1$ in the ranges $0.96 \le x \le 1.6$, $2.2 \le x \le 2.8$ and $3.5 \le x \le 4$.

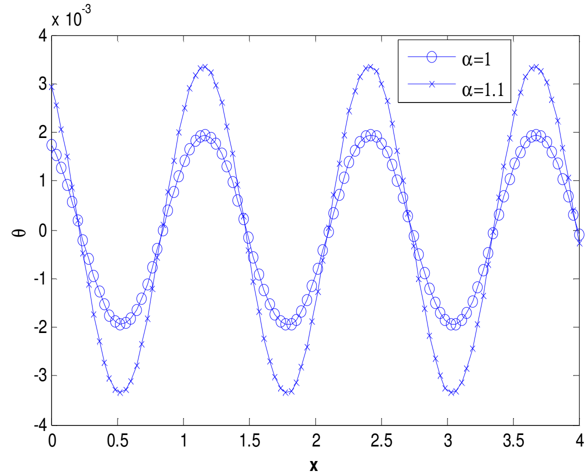

Fig. 3 Variation of the temperature θ versus x

The behavior of the temperature θ for different values of α is shown in Fig. 3. The magnetic field has an increasing effect in the ranges $0.8 \le x \le 1.5$, $2.1 \le x \le 2.7$ and $3.4 \le x \le 4$ while has a decreasing effect in the ranges $0.2 \le x \le 0.8$, $1.5 \le x \le 2.1$ and $2.7 \le x \le 3.4$.

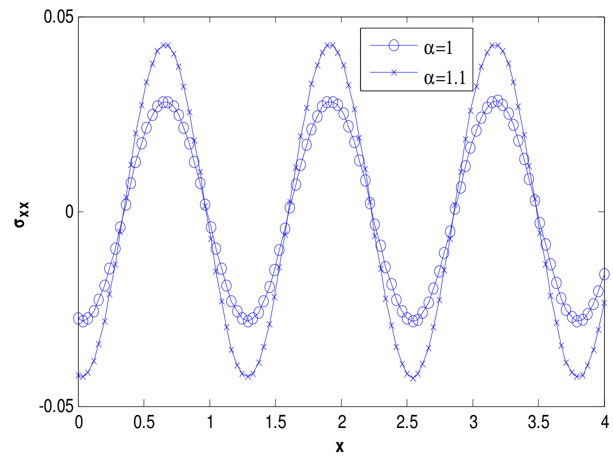

Fig. 4 Variation of the stress tensor σ_{xx} versus x

Fig. 4 represents the variation of the stress tensor σ_{xx}. The values of σ_{xx} for $\alpha = 1.1$ are large compared to the values for $\alpha = 1$ in the ranges $0.3 \le x \le 0.9$, $1.6 \le x \le 2.2$ and $2.9 \le x \le 3.5$, while the values for $\alpha = 1.1$ are smaller than that for $\alpha = 1$ in the ranges $0.9 \le x \le 1.6$, $2.2 \le x \le 2.9$ and $3.5 \le x \le 4$.

Fig. 5 shows the variation of the stress tensor σ_{xz} versus x. The magnetic field has an increasing effect in the ranges $0 \le x \le 0.6$, $1.2 \le x \le 1.9$ and $2.6 \le x \le 3.15$.

It has a decreasing effect in the ranges $0.6 \le x \le 1.2$, $1.9 \le x \le 2.6$ and $3.2 \le x \le 3.8$.

Fig. 6 represents the variation of the stress tensor σ_{zz}. The values of σ_{zz} for $\alpha = 1.1$ are large compared to those for $\alpha = 1$ in the ranges $0.9 \le x \le 1.5$, $2.2 \le x \le 2.8$ and $3.4 \le x \le 4$, but these are small in the ranges $0.3 \le x \le 0.9$, $1.5 \le x \le 2.2$ and $2.8 \le x \le 3.4$.

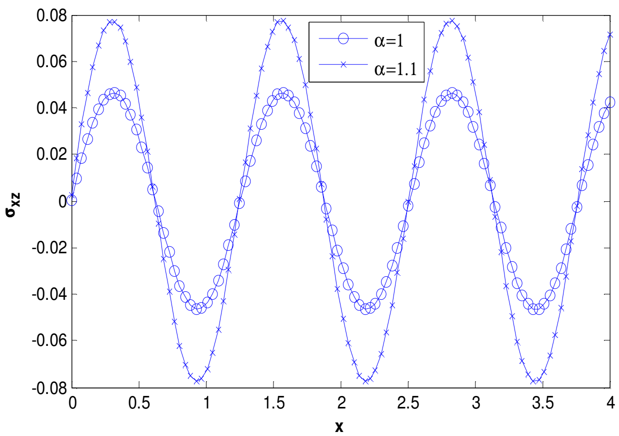

Fig. 5 Variation of the stress tensor σ_{xz} versus x

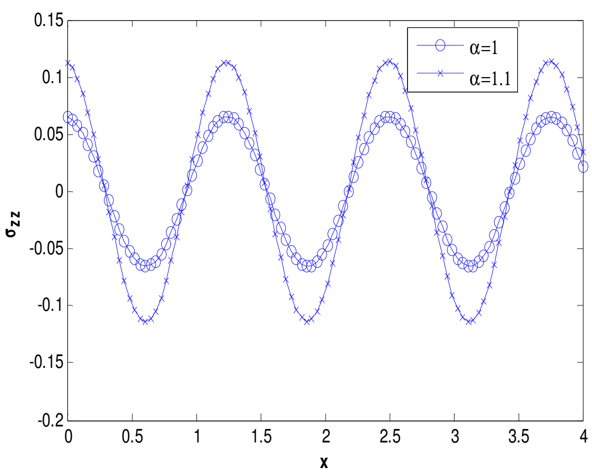

Fig. 6 Variation of the stress tensor σ_{zz} versus x

The behavior of the chemical potential for different values α is exhibited in Fig. 7. The magnetic field has an increasing effect on P the ranges $0.3 \le x \le 0.9$, $1.6 \le x \le 2.2$ and $2.8 \le x \le 3.4$, while it has a decreasing effect in the ranges $0.9 \le x \le 1.6$, $2.2 \le x \le 2.8$ and $3.4 \le x \le 4$.

Fig. 8 depicts the behavior of concentration C. The values of C for $\alpha = 1.1$ are large compared to those for $\alpha = 1$ in the ranges $0.3 \le x \le 0.9$, $1.6 \le x \le 2.2$ and $2.8 \le x \le 3.4$, while the values of C for $\alpha = 1.1$ are smaller than for $\alpha = 1$ in the ranges $0.9 \le x \le 1.6$, $2.2 \le x \le 2.8$ and $3.4 \le x \le 4$.

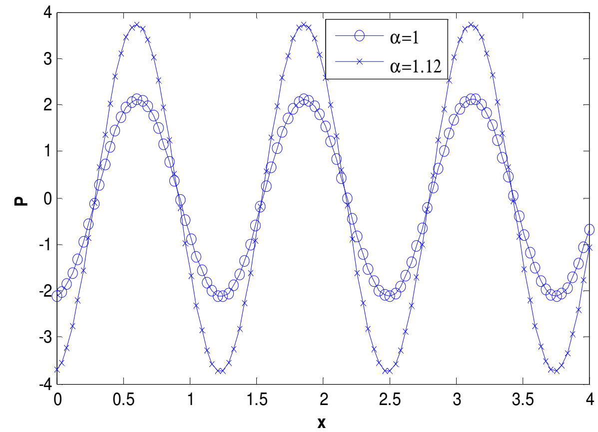

Fig. 7 Variation of the chemical potential P versus x

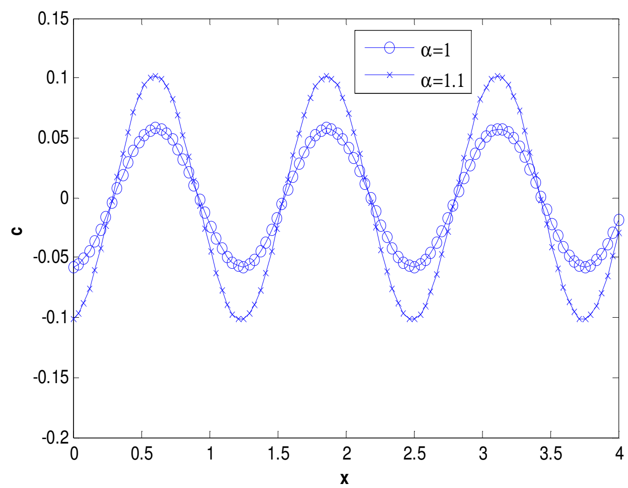

Fig. 8 Variation of the concentration C versus x

V. CONCLUSIONS

The generalized magneto-thermoelastic diffusion with temperature-dependent elastic moduli is discussed. The method of normal mode is used to analyze the problem. It was observed that the magnetic field plays a dual role in the displacement, the temperature, the stress component, the concentration and the chemical potential. This study is very important for micro-scale problems because in this case the material parameters are temperature dependent. The important point of this paper is the consideration that the material properties are dependent on the temperature in the presence of a magnetic field, while in the previous studies the material properties are constant.

VI. References

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Appendix

$$s_1 = \alpha k^2 + \alpha_1 \gamma \omega^2, \qquad s_2 = k^2 + \omega + \tau_0 \omega^2,$$

$$s_3 = \frac{\delta\delta_0}{\alpha_1}(\omega + \tau_0\omega^2), \qquad s_4 = \delta_0(\omega + \tau_0\omega^2) a_1,$$

$$s_5 = \alpha_3(\omega + \tau\omega^2), \qquad m^2 = \frac{\alpha_1\gamma\omega^2}{1-\beta_1} + k^2,$$

$$L_0 = \frac{1}{(\alpha_1\alpha_4\alpha - 1)}[\alpha_1\alpha_4\{s_1 + \alpha(k^2 + s_2)\} - (2k^2 + s_2) + \alpha_1\alpha(s_5 + \alpha_2 s_4) + s_4 + \alpha_1(\alpha_2 + \alpha_4)s_3],$$

$$L_1 = \frac{1}{(\alpha_1\alpha_4\alpha - 1)}[\alpha_1\alpha_4\{k^2(s_1 + s_2\alpha) + s_2 s_1\} + s_5(s_1 + s_2\alpha)\alpha_1 + 2k^2\alpha_1 s_3(\alpha_2 + \alpha_4) + 2s_4 k^2 - k^2(k^2 + 2s_2) + s_5 s_3 - s_4(s_1 + k^2\alpha)\{\alpha_4(1-\alpha_1) - \alpha_1\alpha_2\}],$$

$$L_2 = \frac{1}{(\alpha_1\alpha_4\alpha - 1)}[\alpha_1\alpha_4 s_1 s_2 k^2 - s_2 k^4 + k^4\{\alpha_1(\alpha_2 + \alpha_4)s_3 + s_4\} + \alpha_1 s_2 s_5 s_1 + \alpha_1 s_5 s_3 k^2 + \alpha_1\alpha_2 k^2 s_4 s_1],$$

$$H_i = \frac{s_3(K_i^2 - k^2) + s_4(\alpha K_i^2 - s_1)}{(K_i^2 - s_2 + s_4)},$$

$$R_i = \frac{(\alpha K_i^2 - s_1)(K_i^2 - s_2) - s_3(K_i^2 - k^2)}{(K_i^2 - s_2 + s_4)},$$

$$M_i = \frac{1}{\alpha_1}[K_i^2 - k^2(2\beta_1 - 1) - H_i - R_i],$$

$$M_4 = 2km(\beta_1 - 1), \qquad N_i = 2kK_i, \qquad N_4 = (m^2 + k^2),$$

$$\Delta_1 = K_3 K_2 F^* N_4 (R_3 H_2 - H_3 R_2),$$

$$\Delta_2 = K_1 K_3 F^* N_4 (R_1 H_3 - H_1 R_3),$$

$$\Delta_3 = K_1 K_2 F^* N_4 (R_2 H_1 - H_2 R_1),$$

$$\Delta_4 = F^*[N_1 K_3 K_2 (R_2 H_3 - H_2 R_3) + N_2 K_1 K_3 (R_3 H_1 - H_3 R_1) + N_3 K_1 K_2 (R_1 H_2 - H_1 R_2)],$$

$$\Delta = K_3 K_1 (N_4 M_2 - M_4 N_2)(R_3 H_1 - H_3 R_1) + K_2 K_1 (N_4 M_3 - M_4 N_3)(R_1 H_2 - H_1 R_2) + K_2 K_3 (N_4 M_1 - M_4 N_1)(R_2 H_3 - H_2 R_3).$$

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