Leptons, the subtly Fermions and their Lagrangians for Spinor Fields: Their Integration in the Electromagnetic Strengthening

A-Wollmann Kleinert, Visitant Professor Institute of Physics UNAM, F. Bulnes, Principal of Research TESCHA

Abstract— Considering the Higgs mechanism to re-combine gauge fields of SU(2) and U(1), through three classes of bosons, W, Z, and A, is determined a Lagrangian described through the spinor fields corresponding and by means of the spinorial determination and their integration we obtain a possible representation of the electrical charge associated with the electromagnetism for any particle in the space - time M.

Index Terms—Subtly Fermions, spinorial Lagrangian, electromagnetic strengthening, spinor fields, spin sources

I. INTRODUCTION

In the study of the fermions, any quantum operator Q, acts as translations in fermion dimensions. These translations obtain an electromagnetic energy plus due to the magnetic effects for rotation in phase planes where their Dirac (or Majorana) fields is behave like wave that can be write in the spinor formalism [1]. This property of translation establishes the following conjecture:

“The interaction of fermions shaped for lepton and electron actions inside the space-time M, strengthens the electromagnetic characteristic of any particle of the space-time, that shape their charge”

Likewise, if we consider the state interaction \( Q_{boson} \equiv \int Q_{fermion} \), then in the fermion dimension

a) Unification of gauge couplings  
b) Candidate for dark matter  
c) Non-renormalization theorems  
d) Super-symmetry is broken (that is to say, "1/2" spectrum)  
e) Higgs boson should be found soon  
f) nice new mathematical structures

All this program can be understood like a problem of definition of the background electromagnetic role that exist in the interaction game between three generations leptons where by the Feynman rules and Dirac frame is possible exhibit at least in phase spaces like spinor quantum waves. The spinor context let describe the invariance laws and their direct consequences. In this sense is very useful consider the SM in our treatment. Although also in this work we consider some theoretical experiments that relate the CP aspect and some consequences to SUSY-theory. Also are very useful the topological theory given for the Chern-Simons theory, which only we mentioned as observation gauge theory [2].

In the standard model the three generations leptons are given by the following classification table 1 [3]:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>name</th>
<th>Mass(MeV)</th>
<th>Q(Charge)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>electron</td>
<td>0.511</td>
<td>-1</td>
</tr>
<tr>
<td>( \nu_\mu )</td>
<td>electron neutrino</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \mu )</td>
<td>muon</td>
<td>105.7</td>
<td>-1</td>
</tr>
<tr>
<td>( \nu_\mu )</td>
<td>muon neutrino</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \tau )</td>
<td>tau</td>
<td>1777</td>
<td>-1</td>
</tr>
<tr>
<td>( \nu_\tau )</td>
<td>tau neutrino</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1 MeV = Mega-electron volts = 1.6×10⁻¹⁹ Joule

II. LAGRANGIAN FOR SPINORS

We want to find a suitable Lagrangian for left-and right-handed spinor fields. This could be a:

i) Lorentz invariant and Hermitian  
ii) Quadric in \( \psi_\alpha \) and \( \psi_\alpha^\dagger \), (in this point, equations of motion will be linear with plane wave solutions, which will be suitable for describing free particles)

Then our Lagrangian must involves terms with no derivative
\[ \psi \bar{\psi} = \psi^\dagger \psi \bar{\psi} = e^{ia \bar{\psi} \cdot \psi} \cdot \bar{\psi} \psi, \]

and terms with derivatives:

\[ D \psi = \partial_\mu \psi \partial^\mu \psi, \]

which would lead to a Hamiltonian unbounded from below.

Then to get bounded Hamiltonian, the kinetic term has to contain both \( \psi_a \) and \( \psi^\dagger_a \), where a candidate is:

\[ \sigma \psi = i \psi^\dagger \sigma^\mu \partial_\mu \psi, \]

which is Hermitian up to a total divergence. Indeed,

\[ (\sigma \psi)^\dagger = (i \psi^\dagger \sigma^\mu \partial_\mu \psi)^\dagger = (i \psi^\dagger \sigma^{\mu a} \partial_\mu \psi_a) = -\partial_\mu \psi_a^\dagger (\sigma^{\mu a})^\dagger \psi_a, \]

But all they \( \sigma^{\mu a} \), are Hermitians, that is to say,

\[ \sigma^{\mu a} = (I, -\sigma). \]

Then

\[ (\sigma \psi)^\dagger = -\partial_\mu \psi_a^\dagger (\sigma^{\mu a})^\dagger \psi_a = \partial_\mu \psi_a \sigma^{\mu a} \psi_a^\dagger \]

\[ = i \psi_a \sigma^{\mu a} \partial_\mu \psi_a - i \partial_\mu (\psi_a \sigma^{\mu a} \psi_a) \]

\[ = i \psi^\dagger \sigma^\mu \partial_\mu \psi - i \partial_\mu (\psi^\dagger \sigma^\mu \psi), \]

where the term \( -i \partial_\mu (\psi^\dagger \sigma^\mu \psi) \), dos not contribute to the action. The complete Lagrangian is:

\[ \mathcal{L} = i \psi^\dagger \sigma^\mu \partial_\mu \psi - \frac{1}{2} m \psi \psi^\dagger - \frac{1}{2} m^* \psi^\dagger \psi^\dagger. \]

The phase of \( m \), can be absorbed into the definition of fields, to know:

\[ m = |m| \psi^\dagger, \quad \psi = e^{-ia \bar{\psi} \cdot \psi}, \]

and so without loss of generality we can take \( m \), to be real and positive.

We consider the equation of movement:

\[ -\frac{\partial S}{\partial \psi^\dagger} = -i \partial^\mu \partial_\mu \psi + m \psi^\dagger = 0, \]

and taking Hermitian conjugate (that is to say the transformed equation \(-i \sigma^{\mu a} \partial_\mu \psi + m \psi^\dagger = 0\) we have

\[ + i (\sigma^{\mu a})^\dagger \partial_\mu \psi^\dagger + m \psi_a = 0, \]

But considering \( \sigma^{\mu a} \), are Hermitians, then (7) takes the form

\[ + i \sigma^{\mu a} \partial_\mu \psi^\dagger + m \psi_a = 0, \]

Considering the identity \( \sigma^{\mu a} = e^{a b} e^{a b} \sigma^\mu, \) (8) takes the form:

\[ -i \sigma^{\mu a} \partial_\mu \psi^\dagger + m \psi_a = 0, \]

Using the Yukawa formulation, however we can write:

\[ \mathcal{L}_{\text{Yuk}} = -ye^{i \phi} \psi \bar{\psi}, \psi + \text{h.c.}, \]

(\text{where h.c., is a Hamiltonian constant}) which using the electromagnetic gauge groups to shape the general charge of a particle in the space-time (see the introduction of this work) takes the form

\[ (2,-\frac{1}{2}) \otimes (2,-\frac{1}{2}) \otimes (1,+1) = (1,0) + (3,0), \]

where \( \phi \rightarrow (2,-\frac{1}{2}), \psi \rightarrow (2,-\frac{1}{2}) \) and \( \sigma \rightarrow (1, +1) \). There are not other terms that have mass dimension four or less.

Then in the unitary gauge we have

\[ \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \nu + H(x) & 0 \\ 0 & 0 \end{pmatrix}, \]

and the Yukawa term becomes:

\[ \mathcal{L}_{\text{Yuk}} = - \frac{1}{\sqrt{2}} y(\nu + H)(\ell \bar{\sigma} + \text{h.c.}), \]

which is convenient to label the components of the Lepton doublet as:

\[ \ell = \begin{pmatrix} \nu \\ e \end{pmatrix}, \]

Then the our Yukawa term takes the form
\[ \mathcal{L}_{\text{vay}} = -\frac{1}{\sqrt{2}} \gamma (\nu H) (e \sigma + e^c e^\dagger) \]
\[ = \frac{1}{\sqrt{2}} \gamma (\nu H) \tilde{e} \tilde{e}, \]
from which defines a Dirac field for the electron

\[ \tilde{e} = \begin{pmatrix} e \\ e^\dagger \end{pmatrix}, \]
and we see that electron has acquired a mass

\[ m_e = \frac{\nu v}{\sqrt{2}}, \]
and the neutrino remained massless.

Now consider a theory of two left-handed spinor fields whose Lagrangian comes given for

\[ \mathcal{L} = i \psi_1 \sigma^\mu \partial_\mu \psi_1 - \frac{1}{2} m \psi_1 \psi_1 - \frac{1}{2} m' \psi_1 \psi_1', i = 1, 2, \]  

This Lagrangian is invariant under the actions of group \( SO(2) \), that is to say, of the transformations:

\[ \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \]

Then (18) can be written manifesting the U(1)-symmetry\(^1\) in the form:

\[ \mathcal{L} = i \chi^\dagger \sigma_\mu \partial_\mu \chi + i \xi^\dagger \sigma_\mu \partial_\mu \xi - m^* \chi \xi - m\xi^\dagger \chi^\dagger, \]
where the U(1)-symmetry realizes the correspondences

\[ \chi \rightarrow \exp(-i \theta) \chi, \quad \xi \rightarrow \exp(i \theta) \xi, \]
which are the new rotated spinor fields. Then from (20) the motion equations for this theory comes given in vector form:

\[ \begin{pmatrix} m \partial_\mu \xi^\dagger - i \sigma^\mu \partial_\mu \chi^\dagger \\ -i \sigma^\mu \partial_\mu \xi^\dagger \end{pmatrix} \begin{pmatrix} \chi^\dagger \\ \xi^\dagger \end{pmatrix} = 0, \]

where we can define a four-component Dirac field:

\[ \Psi \equiv \begin{pmatrix} \chi \\ \xi \end{pmatrix}, \]
remaining the Dirac equation

\[ (-i \gamma^\mu \partial_\mu + m) \Psi = 0, \]
But we want to write the Lagrangian in terms of the Dirac field. Let

\[ \Psi^\dagger = (\chi^\dagger, \xi^\dagger), \]
and let’s define

\[ \Psi = \Psi^\dagger \beta = (\xi^\dagger, \chi^\dagger), \]
where \( \beta = \begin{pmatrix} 0 & \delta^a_c \\ \delta^a_c & 0 \end{pmatrix} \) or \( \beta = \gamma^0 \).

Then we find:

\[ \Psi \gamma^\mu \partial_\mu \Psi = \xi^a \chi_a + \chi_a \xi^a, \]
where have

\[ \Psi \gamma^\mu \partial_\mu \Psi = \xi^a \sigma^\mu \partial_\mu \xi^a + \chi_a \sigma^\mu \partial_\mu \chi_a, \]
where using the property \( A \partial B = (\partial A) B + \partial (AB) \), in the term \( \xi^a \sigma^\mu \partial_\mu \xi^a \), we have

\[ \xi^a \sigma^\mu \partial_\mu \xi^a = (\partial_\mu \xi^a) \sigma^\mu \xi^a + \partial_{\mu} (\xi^a \sigma^\mu \xi^a) \]
\[ = (\partial_\mu \xi^a) \sigma^\mu \xi^a + \partial_{\mu} (\xi^a \sigma^\mu \xi^a), \]
and the term \( (\partial_\mu \xi^a) \sigma^\mu \partial_\mu \xi^a \), satisfies

\[ (\partial_\mu \xi^a) \sigma^\mu \partial_\mu \xi^a = +\sigma^\mu \xi^a \partial_\mu \xi^a = +\sigma^\mu \xi^a \partial_\mu \xi^a, \]
In this last step we have used the transformation \( \sigma^{ab} = e^{ab} e^{\dagger b} \). Thus we have

\[ \Psi \gamma^\mu \partial_\mu \Psi = \chi^\dagger \sigma^\mu \partial_\mu \chi + \xi^\dagger \sigma^\mu \partial_\mu \xi + \partial_{\mu} (\xi^a \sigma^\mu \xi^a), \]
Then the Lagrangian can be written as:

\[ \mathcal{L} = i \Psi \gamma^\mu \partial_\mu \Psi - m \Psi \Psi, \]

\(^1\) We consider the transformation given by:

\[ \chi = \frac{1}{\sqrt{2}} (\psi_1 + i \psi_2), \]
\[ \xi = \frac{1}{\sqrt{2}} (\psi_1 - i \psi_2). \]
The $U(1)$-symmetry is deduced immediately:

$$\Psi \to \exp(-i\theta)\Psi, \quad \Psi \to \exp(+i\theta)\Psi,$$

(33)

The Nether current associated with this symmetry is [4]:

$$j^\mu(x) = \frac{\partial L(x)}{\partial (\partial_\mu \varphi_\mu(x))} - \delta_\mu(x),$$

(34)

where

$$j^\mu = \bar{\Psi} \gamma^\mu \psi = \chi^\dagger \varepsilon^\mu \chi - \varepsilon^\dagger \sigma_\mu \varepsilon.$$

(35)

After we will see that (35) is the electromagnetic current, and therefore comes from of a charge that integrates all charges of different three generations of Leptons.

If we want to go back from 4-component Dirac of Majorana fields [4], to the 2-component Weyl fields [5], it is useful to define a projection matrix:

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

(36)

This name of Lepton $\gamma_5$, come from the Fermi theory and this is a singular projection matrix from a source established in the fundamental Lepton given by neutrino + electron + h.c. in a Lagrangian contribution in the pure electromagnetic charge without interaction with other Leptons. Furthermore by the restrictions studied in [4] in presence of the other sources given by the equation

$$\left(1 - i\lambda q \gamma_5 \right)\eta(x) = 0,$$

(37)

we can assume that a source constraint states a universal characteristic of all realistic mechanisms that contribute to the creation or annihilation of a given particle $x(s)$, in the space-time $M^\sharp$.

We can define left and right projection matrices:

$$P_{\text{left}} = \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} \delta_\mu & 0 \\ 0 & 0 \end{pmatrix},$$

(38)

and

$$P_{\text{right}} = \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \delta_\mu \end{pmatrix},$$

(39)

To the Dirac field we have:

$$P_{\text{left}} \Psi = \begin{pmatrix} Z_\mu \\ 0 \end{pmatrix},$$

(40)

and

$$P_{\text{right}} \Psi = \begin{pmatrix} 0 \\ \varepsilon^\mu \end{pmatrix},$$

(41)

where $\Psi$, is given as (23). Then we can describe the neutrino with a Majorana field:

$$\mathcal{N} = \begin{pmatrix} \nu \\ \nu^\dagger \end{pmatrix},$$

(42)

which is convenient to define:

$$\mathcal{N}_{\text{left}} \equiv P_{\text{left}} \mathcal{N} = \begin{pmatrix} \nu \\ 0 \end{pmatrix},$$

(43)

which is a “Dirac field” for the neutrino. Then the kinetic term $i\nu^\dagger \sigma_\mu \partial_\nu \nu$, can be written as $i\mathcal{N}_{\text{left}} \gamma^\mu \mathcal{N}_{\text{left}}^\dagger$.

Now we have that configure out lepton-lepton-gauge boson interaction terms (we want to write covariant derivatives in terms of $W_\mu^\pm$, $Z_\mu$, and $A_\mu$).

We consider the covariant derivatives to electron and lepton respectively [4, 6]:

$$(D_\mu \psi)_i = \partial_\mu \psi - ig_1 (+1)B_\mu \psi,$$

(44)

$$(D_\mu \ell)_i = \partial_\mu \ell_i - ig_2 A_\mu^\ell (T^\nu)_i \ell_j - ig_3 \left(-\frac{1}{2}\right)B_\mu \ell_i,$$

(45)

and the corresponding boson traces:

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^\pm + iA_\mu^\mp),$$

(46)

$$Z_\mu \equiv c_w A_\mu^3 - s_w B_\mu,$$

(47)

$$A_\mu \equiv s_w A_\mu^1 + c_w B_\mu,$$

(48)

then we have

$$g_2 A_\mu T^1 + g_3 A_\mu^2 T^2 = \frac{g_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix},$$

(49)

and

$\eta(x)$ is the Spin $\frac{1}{2}$ source. Also is designed by $\eta_{\zeta(x)}$, where $\zeta_\mu$ is the transposition operator index of spin.
\[ g_z A_\mu^3 T^3 + g_1 B_\mu Y = \frac{e}{s_w} (s_w A_\mu + c_w Z_\mu) T^3 + \]
\[ + \frac{e}{c_w} (c_w A_\mu - s_w Z_\mu) Y \]
\[ = e(A_\mu + \cot \theta_w Z_\mu) T^3 + e(A_\mu - \tan \theta_w Z_\mu) Y \]
\[ = e(T^3 + Y) A_\mu + e(\cot \theta_w T^3 - \tan \theta_w Y) Z_\mu, \]

where by the Gell-Mann-Nishijima formula [7], \((T^3 + Y)\), is the charge \(Q\) calculated from the isopin projection \(T^3\), and weak hypercharge \(Y\) (is possible to demonstrate under integral geometry arguments that the charge live in a orbital space \(\mathbb{M}^4\)), corresponding to the vector bundle \(\mathcal{L}\), using twistor transform on SU(2)-gauges [8] see figure 1).

![Figure 1](image_url)

Then with the charge \(Q = (T^3 + Y)\), the trace \(g_z A_\mu^3 T^3 + g_1 B_\mu Y\), takes the form:

\[ g_z A_\mu^3 T^3 + g_1 B_\mu Y = \]
\[ = e Q A_\mu + e(\cot \theta_w + \tan \theta_w) T^3 - \tan \theta_w Q) Z_\mu \]
\[ = e Q A_\mu + e(T^3 - s_w Q) Z_\mu, \]

(50)

since we have (considering the left-handed Weyl fields for lepton and electron \(\ell \to (2, -1/2), e \to (1, +1)\) and \(\ell = \left( \begin{array}{c} \nu \\ e \end{array} \right) \)

that:

\[ T^3 \nu = + \frac{1}{2} \nu, \quad T^3 e = - \frac{1}{2} e, \quad T^3 \sigma = 0, \]
\[ Y \nu = - \frac{1}{2} \nu, \quad Y e = - \frac{1}{2} e, \quad Y \sigma = + \sigma, \]

(51)

Then the electric charge assignments are as expected:

\[ Q \nu = 0, \quad Q e = - e, \quad Q \sigma = + \sigma, \]

(52)

Likewise the covariant derivatives in terms of the 4-component fields and using (50) are:

\[ (g_z A_\mu^3 T^3 + g_1 B_\mu Y) \mathcal{E} = [ - e A_\mu + e \frac{1}{s_w c_w} ( - \frac{1}{2} P_{\text{Left}} + s_w^2 ) Z_\mu ] \mathcal{E}, \]

(53)

and

\[ (g_z A_\mu^3 T^3 + g_1 B_\mu Y) \mathcal{N}_{\text{Left}} = e \frac{1}{s_w c_w} ( - \frac{1}{2} Z_\mu ) \mathcal{N}_{\text{Left}}, \]

(54)

where \(\mathcal{E}\), and \(\mathcal{N}_{\text{Left}}\), are given by (16) and (43).

Now putting the all pieces together given by (14), (16), (43)-(45), (49), (53)-(54) and the kinetic Lagrangian

\[ \mathcal{L}_{\text{kin}} = i \bar{\nu}^{\mu} \sigma^{\nu} (D_{\mu} \ell), + i \bar{e}^{\mu} \sigma^{\nu} D_{\mu} \sigma, \]

we have the interaction Lagrangian:

\[ \mathcal{L}_{\text{int}} = \frac{1}{\sqrt{2}} g^2 W_{\mu}^{-} j^{-\mu} + \frac{1}{\sqrt{2}} g^2 W_{\mu}^{+} j^{+\mu} + \frac{e}{s_w c_w} Z_{\mu} J_{\mu}^{Z} + e A_\mu J_{\mu}^{EM}, \]

(55)

where we can identify inside (55) the following currents:

\[ J^{+\mu} = \bar{\nu}_{\text{Left}}^{\mu} \sigma^{\nu} \mathcal{N}_{\text{Left}}, \]

(56)

\[ J^{-\mu} = \bar{\mathcal{N}}_{\text{Left}}^{\mu} \sigma^{\nu} \bar{\nu}_{\text{Left}}, \]

(57)

\[ J_{Z}^{\mu} = J_{3}^{\mu} - s_{W}^{2} J_{EM}^{\mu}, \]

(58)

\[ J_{3}^{\mu} = \frac{1}{2} \bar{\mathcal{N}}_{\text{Left}}^{\mu} \mathcal{N}_{\text{Left}} - \frac{1}{2} \bar{\nu}_{\text{Left}}^{\mu} \bar{\nu}_{\text{Left}}, \]

(59)

\[ J_{EM}^{\mu} = - \sigma^{\mu} \sigma, \]

(60)

Now we must connect the before development with the Fermi theory to establish, at least experimentally the charge integration. After through arguments of integral geometry we demonstrate that the integrated charge given by leptons shapes the charge of any quantum particle (like observable) using the Chern–Simons term considering a topological quantum field theory. But before of this, we demonstrate that the process
continued to determine the electrical charges for leptons have a decay proportional beta to the electromagnetic dispersion produced for leptons with masses lower than the bosons $W$, and $Z$. This demonstrates that the load $Q$ only is shaped by the bosons $W_\mu$, $Z_\mu$, and $A_\mu$. The dispersed surplus is a topological part that beta remains free like beta decay and that topologically are remnants of energy of the rotations of the particle like the topological sphere (see the figure 1 (their automorphisms are rotations)) $S^2 \cong SU(2)/U(1)$, [9], being remnants of energy, considering the non-regular leptons under ECS-transformations (see the technical abbreviations in the end of the work) from $SU(2)_{L_y}/U(1)_{y_2}$, whose declarations has been gauged in [9], and where $L_{\vec{s}}$, is the non-regular lepton (non-ordinary), and $Y_{\vec{s}}$, is the swap hypercharge.

Therefore, the automorphism from the $S^2 \cong SU(2)/U(1)$, to itself, which brings the electric charge swap between the muon of tau and neutrino of tau particles, is given by the isometries that form a proper subgroup of the group of projective linear transformations $PGL_2(\mathbb{C}_{(ch_{\text{arg}})})$, namely $PSU_{2}(\mathbb{C}_{\text{arg}})$. Subgroup $PSU_{2}(\mathbb{C})$, is isomorphic to the rotation group $SO(3)_{(\text{ch}_{\text{arg}})}$, [9, 10], which is the isometric group of the unit sphere in three-dimensional real space $\mathbb{R}^3$. The automorphism of the Riemann sphere $\mathbb{C}$, is given by:

$$\text{rot}^{(\text{ch}_{\text{arg}})}_{\text{ECS}} = PSU_{2}^{(\text{ch}_{\text{arg}})} = SO(3)^{(\text{ch}_{\text{arg}})}$$

(61)

where $\mathbb{C}$ is the extended complex plane, $PSU_{2}^{(\text{ch}_{\text{arg}})}$, is the proper subgroup of the projective linear transformations, and swap symmetry, $SO(3)^{(\text{ch}_{\text{arg}})}$, is the group of rotations in 3-dimensional vector space $\mathbb{R}^3$. This can be consigned in the double fibration on a vector bundle of lines $L^2$, in the extended space-time (ad infinitum), that is to say, $\mathbb{C} = C \cup \infty$. The universal cover of $SO(3)_{(\text{ECS})}$ is the special unitary group $SU(2)^{(\text{ECS})}$. This group is also differomorphic to the unit 3-sphere $S^3$.

The answer is in the connection of the Fermi theory. If we want to calculate some scattering amplitude (or a decay rate) of leptons with momentum well below masses of $W$, and $Z$, the propagators effectively become:

$$L_{\text{mass}} = -M_w^2 W^\mu W_\mu - \frac{1}{2} M_Z^2 Z^\mu Z_\mu,$$

(62)

considering the rate $g^{\mu \nu} / M_w^2 W_z$, and the Lagrangian of interaction (55). We get 4-fermion-interaction terms:

$$L_{\text{eff}} = \frac{g^2}{2 M_w^2} J^\mu J_\mu + \frac{e^2}{2 s_w c_w M_Z} J^\mu Z_\mu$$

$$= \frac{e^2}{2 s_w M_w^2} (J^\mu J^-_\mu + J^\mu J^z_\mu)$$

$$= 2 \sqrt{2} G_F (J^\mu J^-_\mu + J^\mu J^z_\mu),$$

(63)

where the Fermi constant is:

$$G_F \equiv \frac{e^2}{4 \sqrt{2} \sin^2 \theta_w M_w^2}.$$

(64)

Generalization to three lepton generations:

$$L_{\text{kin}} = i \ell^\dagger_i \sigma^\mu (D_\mu \ell)_i + i \bar{\ell}^\dagger_i \sigma^\mu D_\mu \ell_i, \quad I = 1, 2, 3$$

Then we can generate the most general Yukawa term of the Lagrangian:

$$L_{\text{yuk}} = -e^{ij} \varphi_i \ell_{IK} y_{IK} \sigma_K + \text{h.c.},$$

(65)

where $y_{IK}$ is a complex $3 \times 3$-matrix (with positive or zero entries on the diagonal) that can be written as unitary bi-transformation $L'^* y\tilde{E}$, where fermion masses are then given by:

$$m_{l_i} = y_{iL} \sqrt{2},$$

(66)

where $y_{iL}$ are diagonal entries. Also from Yukawa term of Lagrangian (65) we have respectively to $\ell_{K}$, and $\sigma_K$, the unitary transformation equations:

$$\ell_i \rightarrow L_{IK} \ell_K,$$

(67)

and

3 The Fermi constants is related with light velocity through the universal constant: $G_F = \frac{\sqrt{2} \alpha}{8 \pi m^2_w} = 1.16637(1) \times 10^{-11} \text{GeV}^{-1}$. 

Fig. 2. Isospin from inner of leptons. This model can be applied in the isomorphism that can be obtained with the field in $SO(3)^{(\text{ch}_{\text{arg}})}$. From a point of view of Chern-Simons theory, this ball degenerates in a many Calabi-Yau manifolds.

How affects this scattering plus in the strengthening of the electromagnetic charge?
\[ \varepsilon_i \rightarrow F_{\mu \nu} \varepsilon_{K} \tag{68} \]

where the currents (56)-(60) remain diagonal, just add the generation index. Then we can calculate decay muon:

\[ \mu^{-} \rightarrow e^{-} \gamma_{\mu} \nu_{\mu} \] \( \tag{69} \)

The moment\( \mu \) for lepton is the mass of lepton and\( g \) is the so-called\( g \)-factor for the lepton. First order approximation quantum mechanics predicts that the\( g \)-factor is 2, for all leptons.

### IV. INTEGRATION OF CHARGE

The Lagrangian of Electromagnetic field coupled to a spinor is given by [13, 14]:

\[ \mathcal{L}_{\text{MAX}} = \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \bar{\psi} \gamma^{\mu} (\partial_{\mu} + ieA_{\mu}) \psi - m \psi \psi, \tag{74} \]

To our case, the Lagrangian of lepton electromagnetic field is (from the above given respectively by\( SU(3)^{\text{charge}} \))

\[ \mathcal{L} = \mathcal{L}_{\text{MAX}}. \]

where the conjugate momentum to the left-handed field is\(^4\):

\[ \pi^{a}(x(s)) = \frac{\partial \mathcal{L}}{\partial (\partial_{0} \psi^{a}_{\mu}(x(s)))} = i \psi^{+}_{a}(x) \sigma_{\alpha a}^{0 a}, \tag{75} \]

and the Hamiltonian is simply given as:

\[ \mathcal{H} = \pi^{a} \partial_{0} \psi^{a} - \mathcal{L} \]

\[ = i \psi^{+}_{a} \sigma_{\alpha a} \psi_{\alpha} - \mathcal{L} \]

\[ = - i \psi^{+}_{a} \sigma_{\alpha a} \partial_{0} \psi_{\alpha} + \frac{1}{2} m (\psi \psi + \psi^{+} \psi^{+}), \tag{76} \]

\(^4\) The momentum variables conjugated, respectively, to\( A_{\alpha} A_{\alpha} \) and\( \psi \).

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\( \varepsilon_i \) to\( F_{\mu \nu} \varepsilon_{K} \)

The currents (56)-(60) remain diagonal, just add the generation index. Then we can calculate decay muon:

\[ \mu^{-} \rightarrow e^{-} \gamma_{\mu} \nu_{\mu} \] \( \tag{69} \)

The moment\( \mu \) for lepton is the mass of lepton and\( g \) is the so-called\( g \)-factor for the lepton. First order approximation quantum mechanics predicts that the\( g \)-factor is 2, for all leptons.

### IV. INTEGRATION OF CHARGE

The Lagrangian of Electromagnetic field coupled to a spinor is given by [13, 14]:

\[ \mathcal{L}_{\text{MAX}} = \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \bar{\psi} \gamma^{\mu} (\partial_{\mu} + ieA_{\mu}) \psi - m \psi \psi, \tag{74} \]

To our case, the Lagrangian of lepton electromagnetic field is (from the above given respectively by\( SU(3)^{\text{charge}} \))

\[ \mathcal{L} = \mathcal{L}_{\text{MAX}}. \]

where the conjugate momentum to the left-handed field is\(^4\):

\[ \pi^{a}(x(s)) = \frac{\partial \mathcal{L}}{\partial (\partial_{0} \psi^{a}_{\mu}(x(s)))} = i \psi^{+}_{a}(x) \sigma_{\alpha a}^{0 a}, \tag{75} \]

and the Hamiltonian is simply given as:

\[ \mathcal{H} = \pi^{a} \partial_{0} \psi^{a} - \mathcal{L} \]

\[ = i \psi^{+}_{a} \sigma_{\alpha a} \psi_{\alpha} - \mathcal{L} \]

\[ = - i \psi^{+}_{a} \sigma_{\alpha a} \partial_{0} \psi_{\alpha} + \frac{1}{2} m (\psi \psi + \psi^{+} \psi^{+}), \tag{76} \]

\(^4\) The momentum variables conjugated, respectively, to\( A_{\alpha} A_{\alpha} \) and\( \psi \).
An appropriate canonical anti-commutation relations are:
\[ \{ \psi_a(x,t), \pi^a(y,t) \} = 0, \]
\[ \{ \psi_a(x,t), \pi^\dagger_a(y,t) \} = i\delta^a_c \delta^3(x-y), \] (77)

where follows update the conjugate momentum (75). Also (77) can take the form:
\[ \{ \psi_a(x,t), \psi_a^\dagger(y,t) \} \sigma^{ sent_c} = \delta^a_c \delta^3(x-y), \] (78)

Using \( \sigma^0 = \sigma^0 = I \), (78) takes the form
\[ \{ \psi_a(x,t), \psi_a^\dagger(y,t) \} = \sigma^{00} \delta^3(x-y), \] (79)

or, equivalently,
\[ \{ \psi_a(x,t), \psi_a^\dagger(y,t) \} = \sigma^{00} \delta^3(x-y), \] (80)

For a four-component Dirac field we found (32), we remember that:
\[ \mathcal{L} = i \bar{\chi} \sigma^\mu \partial_\mu \chi + i \bar{\xi} \sigma^\mu \partial_\mu \xi - m(\bar{\chi} \xi + \bar{\xi} \chi) \]
\[ = i\gamma^\mu \partial_\mu \Psi - m\Psi \Psi, \]

where \( \Psi^\dagger \) and \( \Psi \), are given by (23) and (26). Therefore the corresponding canonical anti-commutation relations are\(^5\):
\[ \{ \Psi_\alpha(x,t), \Psi^\beta(y,t) \} = 0, \]
\[ \{ \Psi_\alpha(x,t), \Psi^\dagger_\beta(y,t) \} = (\gamma^0)_{\alpha\beta} \delta^3(x-y), \] (81)

with the Dirac matrix
\[ \gamma^\mu = \begin{pmatrix} 0 & \sigma^{\mu}_{ac} \\ \sigma^{\mu}_{ac} & 0 \end{pmatrix}, \] (82)

Now using the 4-component Majorana field we found:
\[ \mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{1}{2} m(\bar{\psi} \psi + \psi^\dagger \psi^\dagger) \]
\[ = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} m\bar{\Psi} \Psi \bar{\Psi} \Psi, \]
\[ = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} m\bar{\Psi} \gamma^\dagger \gamma^\mu \gamma^\dagger \Psi \]
\[ = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi - \frac{1}{2} m\bar{\Psi} \gamma^\dagger \gamma^\mu \gamma^\dagger \Psi, \]

where \( \Psi^\dagger = \Psi^\beta = (\psi^a, \psi^a_\dagger) \), or \( \Psi = \Psi^\dagger C \), with

\(^5\) Also can be derived directly from \( \partial \mathcal{L}/\partial (\partial_\mu \Psi) = i\gamma^\mu \gamma^\dagger \Psi \ldots \)

\[ C = \begin{pmatrix} -\epsilon^{ac} & 0 \\ 0 & -\epsilon^*_{ac} \end{pmatrix}, \] (83)

and the corresponding canonical anti-commutation relations are:
\[ \{ \Psi_\alpha(x,t), \Psi^\beta(y,t) \} = (C\gamma^0)_{\alpha\beta} \delta^3(x-y), \]
\[ \{ \Psi_\alpha(x,t), \Psi^\dagger_\beta(y,t) \} = (\gamma^0)_{\alpha\beta} \delta^3(x-y), \] (84)

Now we want to find a electromagnetic manifestation that permit us establish the electromagnetic strengthening of the existence of charge. Field and particle are the same in microscopic theory, also particle and wave are the same in one sense more wide. Then if we can to find the electromagnetic wave comes from the characterized spinor fields who comes from the lepton and electron actions \( \mathcal{L}_e, \mathcal{L}_e \), included inside their Lagrangian, we can have an argument that establish an electromagnetic source with force given by the second member in (84). This force defines the electromagnetic strengthening that require to shape the electric charge of particles \( \chi(s) \) in the space-time \( \mathcal{M} \).

Indeed the previous arguments considered to the spinor equation (37). Then to our spinor \( \Psi \), the Dirac equation
\[ (-i\gamma^\mu + m)\Psi = 0, \] (85)

where we have used the Feynman slash\(^6\):
\[ a = a_\mu \gamma^\mu, \] (86)

then we find:
\[ 0 = (i\gamma^\mu + m)i\gamma^\mu \Psi \]
\[ = (\gamma^\mu + m^2)\Psi \]
\[ = (-\gamma^\mu + m^2)\Psi, \] (87)

which is the wave equation. The Dirac (or Majorana) field satisfies the Klein-Gordon equation and so the Dirac equation has plane-wave solutions! Then the electromagnetic wave nature persists in the lepton interchange like neutrinos, \( \tau \) — particles, muons etc.

Now consider a solution of the form
\[ \Psi(x) = u(p)e^{ipx} + \nu(p)e^{-ipx}, \] (88)

\[ a = a_\mu \gamma^\mu \gamma^\gamma = a_\mu a_\gamma \left( \frac{1}{2} [\gamma^\mu, \gamma^\gamma] + \frac{1}{2} [\gamma^\mu, \gamma^\gamma] \right) \]
\[ = a_\mu a_\gamma \left( -g^{\mu\gamma} + \frac{1}{2} [\gamma^\mu, \gamma^\gamma] \right) = -a_\mu a_\gamma g^{\mu\gamma} + 0 = -a_\mu a_\gamma g^{\mu\gamma}. \]
where \( u(p) \) and \( \nu(p) \) are 4-component constant spinors, and \( p \) is the 4-dimensional momentum \( p^0 = \sqrt{p^2 + m^2} \), plugging it into the Dirac equation gives:

\[
(p + m)u(p)e^{ipx} + (-p + m)\nu(p)e^{-ipx} = 0, \quad (89)
\]

that requires

\[
(p + m)u(p) = 0, \\
(p - m)\nu(p) = 0, \quad (90)
\]

where each equations have two solutions. Then the general solution to the Dirac equation in the spinor wave form is:

\[
\Psi(x) = \sum_{s=\pm} dp [b_s(p)u_s(p)e^{ipx} + d_s^\dagger(p)\nu_s(p)e^{-ipx}], \quad (91)
\]

V. EXPERIMENTS

Remember that the muon decay given in (69) has a charged relevant part characterized by the Lagrangian effective from the Yukawa and kinetic Lagrangians. These decaying involves to the \( \tau^{-} \) decay given by

\[
W^{+}W^{-} \rightarrow qq\tau\nu_{\tau}, \quad (92)
\]

where \( W \), boson can decay leptonically or hadronically.

\[\text{Fig. 6. Kinetic of decaying } W^{+}W^{-} \rightarrow qq\tau\nu_{\tau}. \text{ X, explain the all decay products of the } \tau.\]

Other aspects are observed under the \( \tau^{-} \) decaying, for example in the traces observed inside the Wilson camera and through of the particle physics we can give a 3-D model of the helicities \( \tau^{-} \) leptons group (see figure 7).

\[\text{Fig. 7. All helicity of } \tau^{-} \text{ leptons.}\]

The potential energy obtained by decaying is accumulated in the jet fakes in low energy (see the figure 8). These shape the neutrinos in the background space-time of the \( \text{lepton + photon} \) action. In the imaginary circle could be the bosons \( W^{-} \) that realize the decomposition in \( \tau^{-} \) decaying in the peaks or extremes of the surface \( J^+ \).

\[\text{Fig. 8. Jet fakes in low energy. In the peaks are the considerable } \tau^{-} \text{-decaying. In the middle level we find energy by } \tau_{\gamma}. \text{ Observe the } \tau_{\gamma}, \text{ obtained and angular direction as was showed in figure 6.}\]
The spinor formalism is very useful to obtain properties of the fields like particles from the information of their energy. Furthermore some properties and characteristics of the quantum operators as their conjugation, Hermitian properties, and commutation can be studied under aspects of realization through of path integrals and their transforms [16] using the spinor solutions come from of the Dirac equation and Weyl equation, this last, to the use of the representations of homogeneous classes of Lie groups and their orbital groups in the case that is required a direct integration on fields. From a point of view of the quantum physics the spinor formalism bring the application of similar properties used in the invariants theory, and the motion equations can be linearized like plane wave solutions that is the case of the solution in (91) to leptons. Then the Lagrangians through the spinor formalism can be writing without the added of the mass of the particle explicitly. Then the electromagnetic contribution stays completely exposes and with it we can demonstrate that all interactions that are given with leptons have a minimal decaying which is shaped by the energy plus and their hypercharge. The interchange due to interaction between leptons and electrons produces a energy plus that is useful to strength the charge existence and unity of the charge to every particle in the space-time. This does that always there is electromagnetic energy available to realize the charge transit and their transformations in the space-time.

**TECHNICAL NOTATION AND ABBREVIATIONS**

ECS - Electric charged swap symmetry (in this case to leptons)

SWAP - Symmetry of a Physical System. A family of particular transformations may be continuous (such as rotation of a circle) or discrete (e.g., reflexion of a bilaterally symmetric figure, or rotation of a regular polygon).

QCD – Quantum Chorno-Dynamics.

**CDF** – Chomo-Dynamics of Field.

$\psi^a$ – Spinor field in the covariant notation.

**SM** – Standard Model is a theory concerning the electromagnetic, weak and strong nuclear interactions, which mediate the dynamics of the known subatomic particles. In the Standard Model, the Higgs field is a complex spinor of the group $SU(2)_H$:

$$\psi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \psi^+ \\ \psi^- \end{array} \right)$$

$E_T$ – Equivalent energy of photons

$\psi_I$ – Spinor field in the invariant notation.

**CP** – Charge parity. The CP-violation is a violation of the postulated CP-symmetry (or Charge conjugation Parity symmetry): the combination of $C$-symmetry (charge conjugation symmetry) and $P$-symmetry (parity symmetry). CP-symmetry states that the laws of physics should be the same if a particle were interchanged with its antiparticle ($C$ symmetry), and then left and right were swapped ($P$ symmetry).

**J**$^+$ - Background space-time of the lepton + photon action (jet fakes space).

**REFERENCES**


